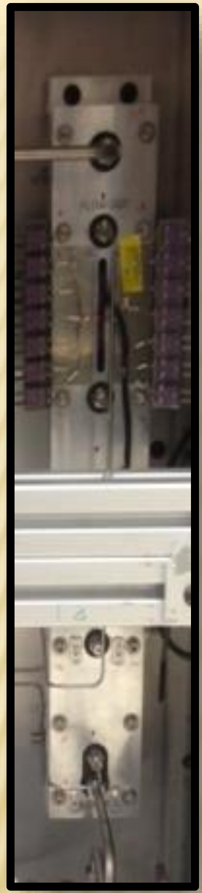

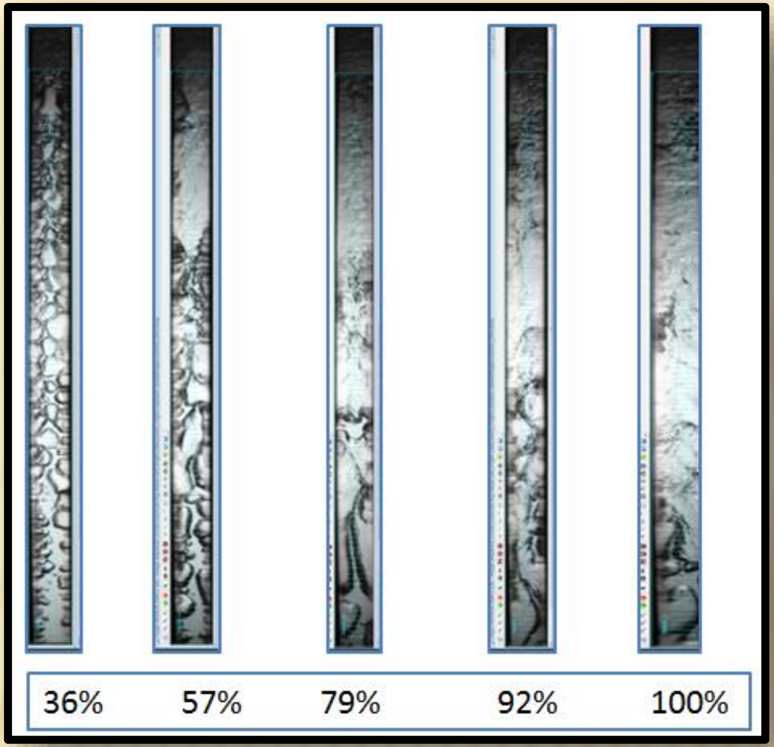


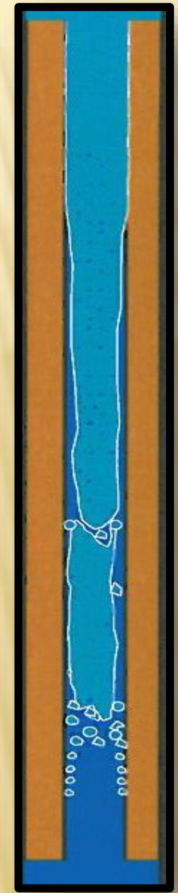


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# TWO PHASE FLOWS AND HEAT TRANSFER

TFAWS- 2016

August 1-August 5, 2016

# TWO PHASE FLOWS AND PHASE TRANSITION

## References


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- 2. Collier J.G and Thome J.R, "Convective Boiling and Condensation", 3rd Edition, Oxford University Press Inc., NY, 2001
- 3. Fox, Pritchard and McDonald, “Introduction to Fluid Mechanics”, 7th Edition.
- 4. Holman, “Heat Transfer”, Fourth Edition, McGraw-Hill Inc., 1976

# OUTLINE

- In this short course, we will address the following topics
  - Two-phase flows hydrodynamics and pressure drop of evaporating and condensing flows
    - Homogeneous Equilibrium Model
    - Separated Flow Model
  - Two-phase flows heat transfer and heat transfer coefficients predictions in evaporating and condensing flow
    - Homogeneous Equilibrium Model
    - Separated Flow Model
- This short course focuses on predictive methods for calculation of two phase pressure drop and heat transfer

# One Dimensional Two Phase Flow

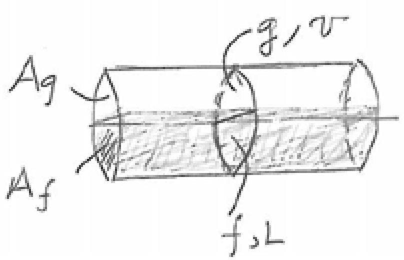
## Definitions of Two-Phase Flow Parameters

- Area

One Dimensional Two-phase Flow

Area $\quad A_f, A_g \quad [m^2]$

$$A = A_f + A_g$$




- Flow Rates

Mass flow rate

$W_g, W_f \quad [kg/s]$

$$W = W_g + W_f$$

Mass Velocity

$$G = \frac{W}{A} \quad kg/m^2.s$$

Volumetric Flow rate.

$Q_g, Q_f$

$$Q = Q_g + Q_f$$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## + Liquid and Gas Phase Velocity

Phase velocity

$u_g, u_f \quad [m/s]$

$$\boxed{u_g = \frac{Q_g}{A_g} = \frac{Q_g \rho_g}{\rho_g \cdot A_g} = \frac{W_g}{\rho_g \cdot A_g}} \; ; \; \boxed{u_f = \frac{Q_f}{A_f} = \frac{W_f}{\rho_f \cdot A_f}}$$

Recall

$$W[kg/s] = Q\left[\frac{m^3}{s}\right] \times \rho\left[\frac{kg}{m^3}\right]$$

## + Volume and Area Based Void Fraction

Void fraction – Volume based

$$\alpha_v = \frac{V_g}{V_f + V_g} =$$

Void fraction – Area Based

$$\alpha = \frac{A_g}{A_g + A_f} = A_g/A$$

Volumetric flow fraction $\beta$.

$$\beta = \frac{Q_g}{Q} = \frac{A_g u_g/u_g}{A_g u_g/u_g + A_f u_f/u_g} = \frac{A_g}{A_g + A_f\left(\frac{1}{S}\right)} \qquad S = u_g/u_f = \text{slip ratio}$$

For most applications $S>1$

For $S=1 \rightarrow$ Homogenous flow $\Rightarrow \beta = \alpha|_{\text{Area-Based}}$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## Flow Quality

Flow Quality

$$x = \frac{W_g}{W_g + W_f} = \frac{\rho_g A_g u_g / \rho_g u_g A_g}{\frac{\rho_g A_g u_g}{\rho_g A_g u_g} + \frac{\rho_f u_f A_f}{\rho_g A_g u_g}} = \frac{1}{1 + \frac{\rho_f}{\rho_g}\frac{A_f}{A_g}\frac{u_f}{u_g}}$$

$$= \frac{1}{1 + \frac{\rho_f}{\rho_g}\frac{A_f}{A_g}\frac{1}{S}} \qquad \frac{A_f}{A_g} = \frac{A - A_g}{A_g} = \frac{A}{A_g} - 1 = \frac{1}{\alpha} - 1 = \left(\frac{1-\alpha}{\alpha}\right)$$

$$\Rightarrow x = \frac{1}{1 + \frac{\rho_f}{\rho_g}\cdot\left(\frac{1-\alpha}{\alpha}\right)\cdot\frac{1}{S}}$$

$$x = \frac{1}{1 + \frac{\rho_f}{\rho_g}\frac{A_f}{A_g}\frac{u_f}{u_g}} \Rightarrow x + x\frac{\rho_f}{\rho_g}\cdot\frac{A_f}{A_g}\frac{u_f}{u_g} = 1$$

$$\frac{A_f}{A_g} = \frac{1-x}{x\frac{\rho_f}{\rho_g}\frac{u_f}{u_g}} = \frac{1}{\alpha} - 1 \Rightarrow \frac{1}{\alpha} = 1 + \frac{1-x}{x\frac{\rho_f}{\rho_g}\frac{u_f}{u_g}}$$

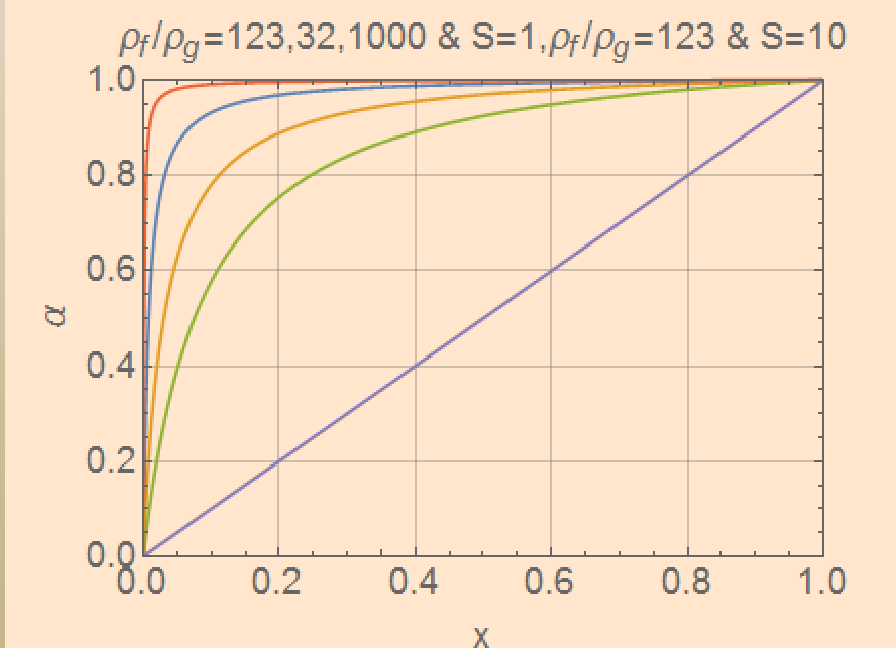
$$\Rightarrow \alpha = \frac{1}{1 + \left(\frac{1-x}{x}\right)\frac{\rho_g}{\rho_f} S} \Rightarrow \text{For a small } x \Rightarrow \text{Large } \alpha$$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## + Calculation of Void Fraction from Flow Quality

```
α[x_, ρf_, ρg_, s_] := 1 / (1 + ((1-x)/x) (ρg/ρf) s)

Plot[{α[x, 1.6, .013, 1], α[x, 1.6, .05, 1], α[x, 1.6, .013, 10], α[x, 1, .001, 1], x}, {x, 0, 1}, Frame -> True, PlotRange → {{0, 1}, {0, 1}},
 FrameLabel → {"x", "α", "ρf/ρg=123,32,1000 & S=1,ρf/ρg=123 & S=10", ""}, LabelStyle → (FontSize → 18), AspectRatio → .7, GridLines → Automatic]
```




$$\alpha = \frac{1}{1+\left(\frac{1-x}{x}\right)\frac{\rho_g}{\rho_f}S}$$

Void fraction as a function of quality for different density ratios and slip factors

## + Average Density and Specific Volume of Mixture

Mixture Density $\bar{\rho}$

$$\bar{\rho} = \frac{A_g}{A}\rho_g + \frac{A_f}{A}\rho_f = \alpha\rho_g + (1-\alpha)\rho_f \quad \text{But } \alpha = \left(1+\frac{\rho_g}{\rho_f}\left(\frac{1-x}{x}\right)\right)^{-1} \text{ for } S=1$$

$$\frac{1}{\bar{\rho}} = \frac{1}{\alpha\rho_g + (1-\alpha)\rho_f} \qquad \alpha = \frac{1\cdot x\cdot\rho_f}{\rho_f x + \rho_g(1-x)}$$

$$\frac{1}{\bar{\rho}} = \frac{1}{\frac{x\rho_f\cdot\rho_g}{x\rho_f+\rho_g(1-x)} + \left(1-\frac{x\rho_f}{x\rho_f+\rho_g(1-x)}\right)\cdot\rho_f}$$

$$= \frac{1}{\frac{x\rho_f\rho_g}{x\rho_f+\rho_g(1-x)} + \frac{\rho_f\left(x\rho_f+\rho_g(1-x)-x\rho_f\right)}{x\rho_f+\rho_g(1-x)}}$$

$$= \frac{x\rho_f+\rho_g(1-x)}{x\rho_f\rho_g + \cancel{x\rho_f^2} + \rho_f\rho_g(1-x) - \cancel{x\rho_f^2}}$$

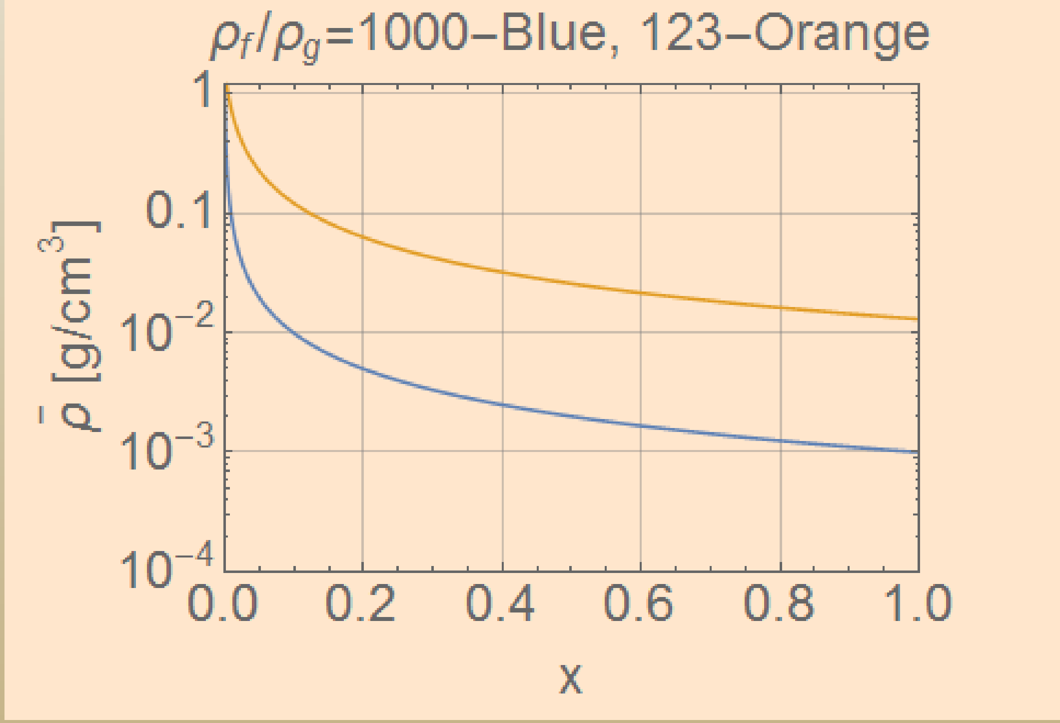
$$= \frac{x\rho_f+\rho_g(1-x)}{x\rho_f\rho_g+\rho_f\rho_g(1-x)} = \frac{x\rho_f+\rho_g(1-x)}{\rho_f\rho_g} = \frac{x}{\rho_g} + \frac{1-x}{\rho_f}$$

$$= x v_g + (1-x)v_f = \bar{v}$$

$$\boxed{\bar{v} = (1-x)v_f + x v_g = 1/\bar{\rho}}$$

# Calculation of Average Density of Mixture

```
ρavg[x_, ρf_, ρg_] := 1 / (x / ρg + (1 - x) / ρf);
LogPlot[{ ρavg[x, 1., .001], ρavg[x, 1.6, .013]}, {x, 0, 1}, Frame -> True, PlotRange → {{0.0001, 1}, {0.0001, 1.2}},
 FrameLabel → {"x", "ρ̄ [g/cm^3]", "ρf/ρg=1000-Blue, 123-Orange", ""}, LabelStyle → (FontSize → 24), AspectRatio → .7, GridLines → Automatic]
```



- Two Phase Flow Regime in a Heated Tube
- Let's define the flow and thermodynamic quality

$Q_f$ = constant

$q''$ = constant

Flow quality

$$x = \frac{\rho_g u_g A_g}{\rho_g u_g A_g + \rho_f u_f A_f} = \frac{W_g}{W}$$

Thermodynamic equilibrium quality

"Mixing Cup" Quality

$$x_e = \frac{h - h_f}{h_{fg}}$$

Mixture enthalpy

**Attributes of thermodynamic quality**

If $h < h_f \Longrightarrow x_e < 0$
Subcooled Liquid

If $h > h_g \Longrightarrow x_e > 1$
Superheaterd Vapor

# Homogeneous Two-Phase Equilibrium Model

# Homogeneous Two-Phase Equilibrium Model

For the Homogeneous Equilibrium flow

$S=1 \implies x = x_e$ for $0 \le x_e \le 1$

$x \neq xe$ because of the superheated liquid layer near the wall

Homogeneous Two Phase Flow Model

Applicability to Bubble and Mist Flow

## Assumptions

- Uniform Velocity $u_g = u_f = u \Rightarrow S=1$
- Uniform pressure $p_g = p_f = p$

Homogenous equilibrium Model

$$T_g = T_f = T_{sat}\big|_{p=\text{local pressure}} \Rightarrow x_e = x \quad 0 \le x_e \le 1$$

# Conservation Laws and the Laws of Thermodynamics

## Conservation of Mass

$$\left.\frac{dM}{dt}\right|_{System} = 0$$

$$M_{System} = \int_{M_{System}} dm = \int_{V_{System}} \rho dv$$

## Newton's Second Law

$$\left.\frac{d\vec{P}}{dt}\right|_{System} = \sum \vec{F}$$

$$\vec{P}_{System} = \int_{M_{System}} \vec{V} dm = \int_{Vol_{System}} \vec{V} \rho dv$$

## The First and Second Law of Thermodynamics

$$\left.\frac{dE}{dt}\right|_{System} = \dot{Q} - \dot{W}$$

$$E_{System} = \int_{M_{System}} e dm = \int_{V_{System}} e \rho dv$$

$$e = u + \frac{V^2}{2} + gz$$

$$\left.\frac{dS}{dt}\right|_{System} \geq \frac{1}{T} \dot{Q}$$

$$S_{System} = \int_{M_{System}} s dm = \int_{V_{System}} s \rho dv$$

- Develop a Control Volume Formulation from system rate relations
- For any extensive property N representing mass, momentum, energy, entropy or angular momentum,
  - Then, for any extensive property

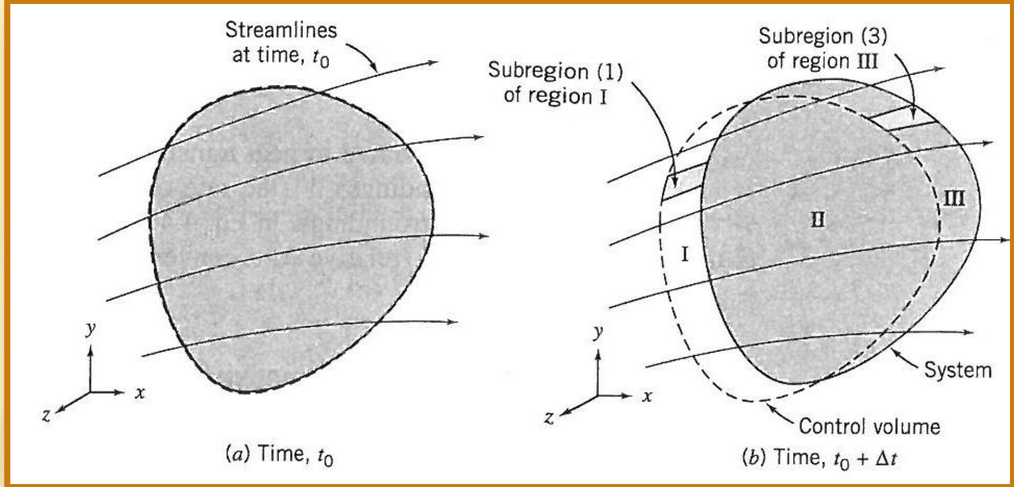
$$
\begin{array}{ll}
N = M, & \text{then } \eta = 1 \\
N = \vec{P}, & \text{then } \eta = \vec{V} \\
N = \vec{H}, & \text{then } \eta = \vec{r} \times \vec{V} \\
N = E, & \text{then } \eta = e \\
N = S, & \text{then } \eta = s
\end{array}
$$

  - The following is true…

$$N_{\text{system}} = \int_{M(\text{system})} \eta \, dm = \int_{\forall(\text{system})} \eta \, \rho \, d\forall$$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

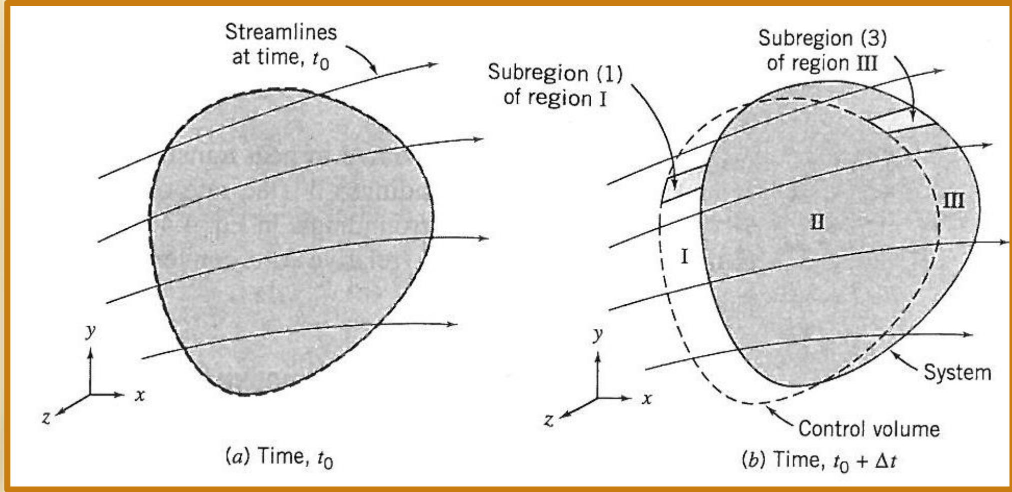
- Select an arbitrary piece of fluid flowing at $t=t_0$
- Die the piece in blue and take the shape of this piece as the control volume which is fixed in space




- After $\Delta t$, the system has moved to another point in space yet the control volume is still fixed in space.
- Examining the control volume/system pair geometry at $t_0$ and $t_0 + \Delta t$ will result in the control volume description

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

- At $t=t_0$, the system is in the control volume.
- At $t= t_0 + \Delta t$, part of the system is in the control volume
- Three regions result. They are labeled as regions I, II, III




$$\left.\frac{dN}{dt}\right]_{System} = \lim_{\Delta t \to 0} \frac{N_S\big]_{t_0+\Delta t} - N_S\big]_{t_0}}{\Delta t} \longrightarrow N_s)_{t_0} = (N_{CV})_{t_0} \quad +$$

$$N_s)_{t_0+\Delta t} = (N_{II} + N_{III})_{t_0+\Delta t} = (N_{CV} - N_I + N_{III})_{t_0+\Delta t}$$

$$\left.\frac{dN}{dt}\right)_s = \lim_{\Delta t \to 0} \frac{(N_{CV} - N_I + N_{III})_{t_0+\Delta t} - N_{CV})_{t_0}}{\Delta t}$$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

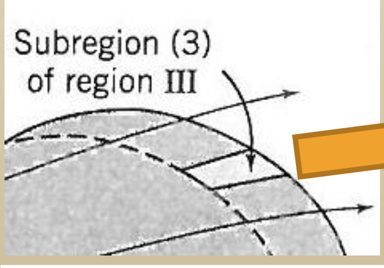
Since the limit of a sum is equal to the sum of the limits, we can write

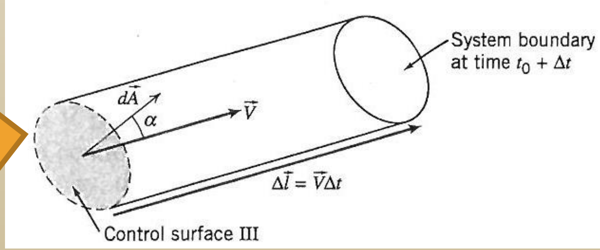
$$\left.\frac{dN}{dt}\right)_s = \underbrace{\lim_{\Delta t\to 0}\frac{N_{\mathrm{CV}})_{t_0+\Delta t} - N_{\mathrm{CV}})_{t_0}}{\Delta t}}_{①} + \underbrace{\lim_{\Delta t\to 0}\frac{N_{\mathrm{III}})_{t_0+\Delta t}}{\Delta t}}_{②} - \underbrace{\lim_{\Delta t\to 0}\frac{N_{\mathrm{I}})_{t_0+\Delta t}}{\Delta t}}_{③}$$

Identify the three regions

For region III-Integration over the subregions3

$$\lim_{\Delta t\to 0}\frac{N_{\mathrm{CV}})_{t_0+\Delta t} - N_{\mathrm{CV}})_{t_0}}{\Delta t} = \frac{\partial N_{\mathrm{CV}}}{\partial t} = \frac{\partial}{\partial t}\int_{\mathrm{CV}} \eta\,\rho\,d\forall$$



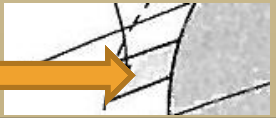

$$dN_{\mathrm{III}})_{t_0+\Delta t} = (\eta\,\rho\,d\forall)_{t_0+\Delta t}$$

$$d\forall = \Delta l\,dA\cos\alpha = \Delta\vec{l}\cdot d\vec{A} = \vec{V}\cdot d\vec{A}\,\Delta t$$

$$\lim_{\Delta t\to 0}\frac{N_{\mathrm{III}})_{t_0+\Delta t}}{\Delta t} = \lim_{\Delta t\to 0}\frac{\int_{\mathrm{CS_{III}}} dN_{\mathrm{III}})_{t_0+\Delta t}}{\Delta t} = \lim_{\Delta t\to 0}\frac{\int_{\mathrm{CS_{III}}} \eta\,\rho\vec{V}\cdot d\vec{A}\,\Delta t}{\Delta t} = \int_{\mathrm{CS_{III}}} \eta\,\rho\vec{V}\cdot d\vec{A}$$

For region I-Integration over the sub-regions 1

Subregion (1)
of region I



$$\lim_{\Delta t\to 0}\frac{N_{\mathrm{I}})_{t_0+\Delta t}}{\Delta t} = -\int_{\mathrm{CS_I}} \eta\,\rho\vec{V}\cdot d\vec{A}$$

- This yields

$$\left.\frac{dN}{dt}\right)_{\text{system}} = \frac{\partial}{\partial t}\int_{\text{CV}} \eta\,\rho\,d\forall + \int_{\text{CS}_{\text{I}}} \eta\,\rho\vec{V}\cdot d\vec{A} + \int_{\text{CS}_{\text{III}}} \eta\,\rho\vec{V}\cdot d\vec{A}$$

- Since $\text{CS}_{\text{I}}$ and $\text{CS}_{\text{III}}$ form the Control Volume Surface, the last two surface integrals combine into one integral namely,

$$\left.\frac{dN}{dt}\right)_{\text{system}} = \frac{\partial}{\partial t}\int_{\text{CV}} \eta\,\rho\,d\forall + \int_{\text{CS}} \eta\,\rho\vec{V}\cdot d\vec{A}$$

- ***Reynolds Transport Theorem*** relates the rate of change of the extensive property N of the system with the variation of the property associated with the control volume

# Physical Interpretation

| | |
|---|---|
| $\left.\frac{dN}{dt}\right)_{\text{system}}$ | is the rate of change of the system extensive property $N$. For example, if $N = \vec{P}$, we obtain the rate of change of momentum. |
| $\frac{\partial}{\partial t}\int_{\text{CV}} \eta\, \rho\, d\forall$ | is the rate of change of the amount of property $N$ in the control volume. The term $\int_{\text{CV}} \eta\, \rho\, d\forall$ computes the instantaneous value of $N$ in the control volume ($\int_{\text{CV}} \rho\, d\forall$ is the instantaneous mass in the control volume). For example, if $N = \vec{P}$, then $\eta = \vec{V}$ and $\int_{\text{CV}} \vec{V} \rho d\forall$ computes the instantaneous amount of momentum in the control volume. |
| $\int_{\text{CS}} \eta\, \rho \vec{V} \cdot d\vec{A}$ | is the rate at which property $N$ is exiting the surface of the control volume. The term $\rho\vec{V} \cdot d\vec{A}$ computes the rate of mass transfer leaving across control surface area element $d\vec{A}$; multiplying by $\eta$ computes the rate of flux of property $N$ across the element; and integrating therefore computes the net flux of $N$ out of the control volume. For example, if $N = \vec{P}$, then $\eta = \vec{V}$ and $\int_{\text{CS}} \vec{V} \rho \vec{V} \cdot d\vec{A}$ computes the net flux of momentum out of the control volume. |

## HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

# One Dimensional Conservation of Mass, Momentum and Energy

## Conservation of Mass

$$\left.\frac{dM}{dt}\right|_{System} = 0$$

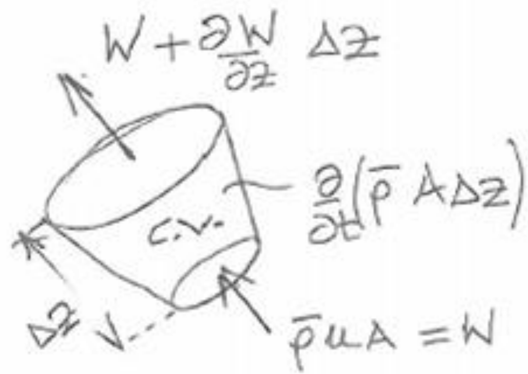
$$M_{System} = \int_{M_{System}} dm = \int_{V_{System}} \rho dv$$

Conservation of Mass

Transport theorem

$$\underbrace{\frac{\partial}{\partial t}\int_{V} \bar{\rho}\, dv}_{\text{storage rate}} + \underbrace{\int_{S} \bar{\rho}\, \vec{u}_r \cdot d\vec{S}}_{\text{Mass out} - \text{Mass in}} = 0$$

$$\frac{\partial}{\partial t}\left(\bar{\rho} A \Delta z\right) + \cancel{W} + \frac{\partial W}{\partial z}\Delta z - \cancel{W} = 0$$



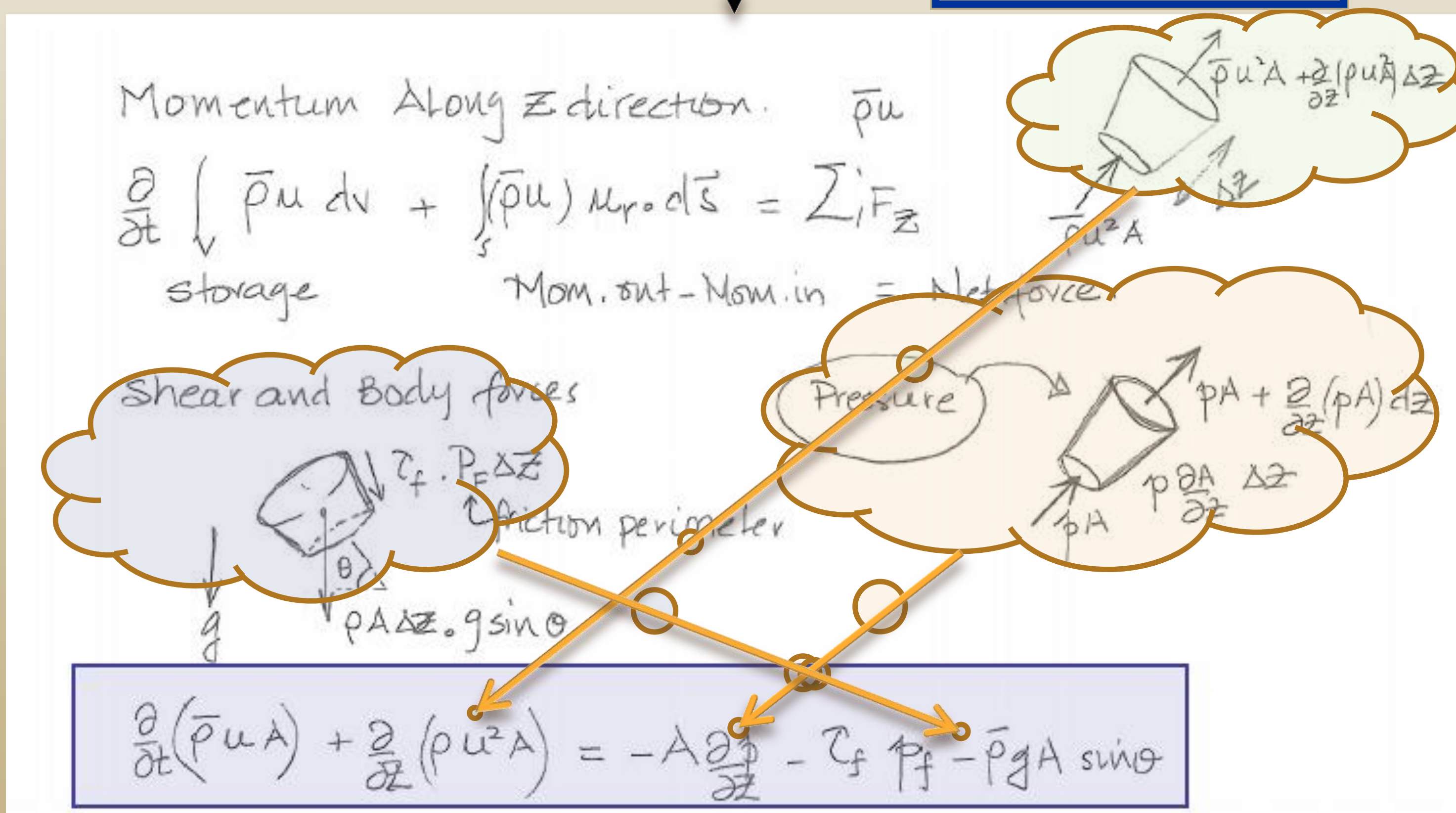

$$\boxed{\frac{\partial}{\partial t}\left(\bar{\rho} A\right) + \frac{\partial W}{\partial z} = 0}$$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## + Conservation of Momentum

$$\left.\frac{d\vec{P}}{dt}\right|_{System} = \sum \vec{F}$$

$$\vec{P}_{System} = \int_{M_{System}} \vec{V}dm = \int_{Vol_{System}} \vec{V}\rho dv$$



# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## + Conservation of Energy

$$\left.\frac{dE}{dt}\right|_{System} = \dot{Q} - \dot{W}$$

$$E_{System} = \int_{M_{System}} e\,dm = \int_{V_{System}} e\rho\,dv$$

$$e = u + \frac{V^2}{2} + gz$$

$$\frac{\partial}{\partial t}\left(\bar{\rho}h^0 A\right) + \frac{\partial}{\partial z}\left(\bar{\rho}h^0 uA\right) = q'' P_H + q''' A + \frac{\partial}{\partial t}(pA) \quad where\ h^0 = e^0 + pv$$

Conservation of energy.— $e^0 = e + \frac{u^2}{2} + gz\sin\theta$

$$\frac{J}{kg} = \frac{kg\frac{m}{s^2}\times m}{kg} \rightsquigarrow \frac{m^2}{s^2} \checkmark$$

Internal energy storage rate + Internal Energy out − In = Rate of Heat transferred to C.V. − Rate of work done by C.V.

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## + Assumptions

Steady State $\frac{\partial}{\partial t} = 0$ ; No chem. reaction $q''' = 0$

Tubular geometry $A = \text{const}$.

Neglect changes in kinetic and potential energy
Assume constant properties for individual phases

$$\frac{\partial}{\partial t}(\bar{\rho} A) + \frac{\partial W}{\partial z} = 0 \Rightarrow \boxed{\begin{array}{l} W = \text{const} = \bar{\rho} u A = G A \\ \text{Const. Area} \Rightarrow G = \text{constant} \end{array}}$$

Momentum

$$\frac{\partial}{\partial t}\left(\bar{\rho} u A\right)^{\nearrow 0} + \frac{\partial}{\partial z}\left(\bar{\rho} u^2 A\right) = -A \frac{\partial p}{\partial z} - \tau_F P_F - \bar{\rho} g A \sin\theta \; ; \; \bar{\rho} u A = G \cdot A = W$$

$$\boxed{W \frac{du}{dz} = -A \frac{dp}{dz} - \tau_F P_F - \bar{\rho} A g \sin\theta}$$

Energy

$$\cancel{\frac{\partial}{\partial t}\left(\bar{\rho} h^\circ A\right)}^{\nearrow 0} + \frac{\partial}{\partial z}\left(\bar{\rho} h^\circ u A\right) = q'' P_H + q''' A + \cancel{\frac{\partial}{\partial t}(pA)}^{\nearrow 0}$$

$$\boxed{\Rightarrow W \frac{dh}{dz} = P_H q''}$$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

+ Observations for adiabatic single phase and two phase flows

$1\phi$ For Adiabatic Flow $u = \text{const.}$

" Flow w. Heat transfer $u = \text{const}$

$2\phi$ For Adiabatic Flow $u = \text{const.}$

" Flow w. Heat transfer Boiling → Flow accelerates

Condensation → Flow Decelerates

$\boxed{\begin{array}{l} \text{Remember } \bar{\rho} u = G \equiv \text{constant.} \\ \text{Boiling} \quad x \nearrow, \ \alpha \nearrow, \ \bar{\rho} \searrow \ \Rightarrow \ u \uparrow \\ \text{Condensation} \quad x \searrow, \ \alpha \searrow, \ \bar{\rho} \nearrow \ \Rightarrow \ u \searrow \end{array}}$

# Solution

- Solve the conservation of mass and energy first to obtain the thermodynamic quality

$$\boxed{\frac{\partial}{\partial t}(\bar{\rho}A)+\frac{\partial W}{\partial z}=0}$$

$$+$$

$$\boxed{\frac{\partial}{\partial t}\left(\bar{\rho}h^{0}A\right)+\frac{\partial}{\partial z}\left(\bar{\rho}h^{0}uA\right)=q''P_{H}+q'''A+\frac{\partial}{\partial t}(pA)}$$

$$\Rightarrow \quad \boxed{x}$$

# Solution

- After quality is solved for, solve the momentum equation to obtain the pressure drop

$$\boxed{x}$$

$$+$$

$$\boxed{\frac{\partial}{\partial t}\left(\bar{\rho}uA\right)+\frac{\partial}{\partial z}\left(\bar{\rho}u^{2}A\right)=-A\frac{\partial P}{\partial z}+-\tau_{f}P_{f}-\bar{\rho}gA\sin(\theta)}$$

$$\Rightarrow \quad \boxed{\Delta P}$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

# Solution

Mass + Energy → $x$ → Momentum → $\Delta p$

$$\frac{dh}{dz} = \frac{q'' P_H}{W} \qquad h = h_f + x_e h_{fg}$$

$$h_{fg}\frac{dx_e}{dz} = \frac{q'' P_H}{W} \Rightarrow dx_e = \frac{q'' P_H}{W h_{fg}} dz$$

$$\Rightarrow x_e = x_{e,i} + \frac{P_H}{W h_{fg}} \int_0^z q'' d\zeta$$

What is $x_{e,i}$ $\qquad$ $x_{e,i}$ Thermodynamic Equilibrium quality at inlet

$$x_{e,i} = \frac{h_i - h_f}{h_{fg}} = \frac{\overbrace{-(h_f - h_i)}^{\Delta h_{sub,i}}}{h_{fg}}$$

$$= -\frac{c_{p,f}(T_{sat} - T_i)}{h_{fg}} = -\frac{c_{p,f}\,\Delta T_{sub,i}}{h_{fg}}$$

$$\Rightarrow \boxed{x_e = -\frac{c_{p,f}\,\Delta T_{sub}}{h_{fg}} + \frac{P_H}{W h_{fg}} \int_0^z q'' d\zeta}$$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## Quality and thermodynamic quality

$$h_i = h_f \Rightarrow x_{e,i} = 0$$

$$h_f < h_i < h_g \Rightarrow 0 < x_{e,i} < 1$$

$$h_i = h_g \Rightarrow x_{e,i} = 1$$

$$h_i > h_g \Rightarrow x_{e,i} > 1$$

For superheated region

$$x_{e,i} = \frac{h_i - h_f}{h_{fg}} = \frac{h_g - h_f + h_i - h_g}{h_{fg}} = \frac{h_{fg} + C_{p,g}(T_i - T_{sat})}{h_{fg}}$$

$$= 1 + \frac{C_{p,g}(T_i - T_{sat})}{h_{fg}}$$

$$x = \begin{cases} 0 & x_e < 0 \\ x_e & 0 \le x_e \le 1 \\ 1 & x_e > 1 \end{cases}$$

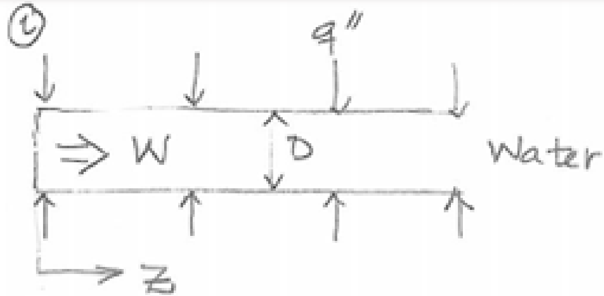
# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## For a uniformly heated circular tube, find the axial location z at which $x_e = 0$ and $x_e = 1$

$T_i < 100°C.$

$$x_e(z) = -\frac{c_{p,f}\,\Delta T_{sub,i}}{h_{fg}} + \frac{P_H}{W h_{fg}} \int_0^z q'' \, dz$$

$$= -\frac{c_{p,f}\,\Delta T_{sub}}{h_{fg}} + \frac{\pi D}{W h_{fg}} q'' z$$




$T_i = 90°C$

$p_i = 1\,atm.$

$$\boxed{x_e = 0 \Rightarrow z\big|_{x_e=0} = \frac{\dfrac{c_{p,f}\cdot\Delta T_{sub}}{\cancel{h_{fg}}}}{\dfrac{\pi D q''}{W \cancel{h_{fg}}}} = \frac{W c_{p,f}\,\Delta T_{sub}}{\pi D q''}}$$

- Uniformly heated Circular Tube
- Finding x(z), α(z), u(z)

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*Homogeneous Two-Phase Flow Model – Steady State Solutions*

| Region | $x$ | $\alpha$ | $u$ |
|---|---|---|---|
| **Subcooled** $z < z\|_{x_e=0}$ | $0$ | $0$ | $\dfrac{G}{\rho_f}$ |
| **Saturated** $z\|_{x_e=0} < z < z\|_{x_e=1}$ | $x_e$ | $\dfrac{1}{1+\dfrac{\rho_g}{\rho_f}\left(\dfrac{1-x_e}{x_e}\right)}$ | $G\left[x_e v_g + (1-x_e)v_f\right]$ |
| **Superheated** $z > z\|_{x_e=1}$ | $1$ | $1$ | $\dfrac{G}{\rho_g}$ |

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# Example Problem Water Upward Flow in a Heated Pipe…

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*Numerical Example 1: Determination of Pressure Drop using HEM with Constant Two-Phase Friction Factor for Heated Vertical Upflow with Saturated Inlet*

Saturated water ($x_e = 0$) at mass velocity $G = 250$ kg/m².s and inlet pressure of $p_i = 5$ bar enters a vertical circular tube of diameter $D = 1$ cm and length $L = 100$ cm, where it is subjected to a constant heat flux $q'' = 10^6$ W/m². Neglecting any kinetic or potential energy effects and assuming constant thermophysical properties, use the Homogeneous Equilibrium Model (HEM) with a constant two-phase friction factor $f_{TP} = 0.003$ to determine the following:

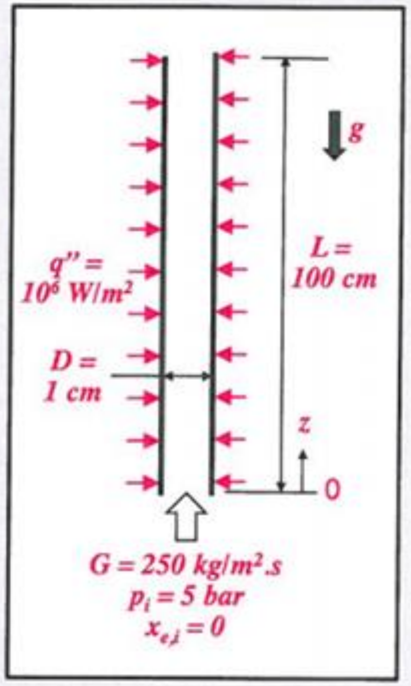
(a) $x_e(z)$, $x_{e,L}$

(b) $\Delta p_F$

(c) $\Delta p_A$

(d) $\Delta p_G$

(e) $\Delta p$

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Course: Two-Phase Flow

Prof. Issam Mudawar

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

- Finding $x_e[z]$ and the z location where the thermodynamic quality $x_e = 0$ $and$ $x_e = 1$
- Finding $x_e[L]$
- Finding x[z] based on $x_e[z]$, and finding x'[z]

$$\texttt{xe[z\_] := } -\frac{\text{Cpf }\Delta\text{Tsub}}{\text{hfg}} + \frac{\pi\text{ DD q}}{\text{W hfg}}\, z$$

$$\texttt{zxe0 = } \frac{\text{W Cpf }\Delta\text{Tsub}}{\pi\text{ DD q}};$$

```
Print["z|xe=0 is ", zxe0]
```

```
z|xe=0 is 0.
```

```
xe[1]; Print["xe[L]=", xe[L]]
```

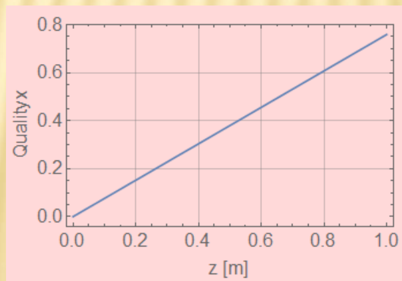
```
xe[L]=0.759013
```

$$\texttt{zxe1 = } \frac{\text{hfg W}}{\pi\text{ DD q}} + \frac{\text{Cpf }\Delta\text{Tsub W}}{\pi\text{ DD q}};$$

```
If[zxe1 > L, zxe1 = L, zxe1];
```

```
p = 5 (*bar*);
Cpf = 4312 (*J/kg.K*);
hfg = 2.108 × 10^6 (*J/kg*);
vf = .0011 (*m^3/kg*);
vg = .3748 (*m^3/kg*);
μf = 180.1 × 10^-6 (*kg/m.s*); μg = 14.06 × 10^-6 (*kg/m.s*);
q = 1.0 × 10^6 (*W/m^2*); ΔTsub = 0 (*°C*);
g = 9.8 (*m.s^-2*);
θ = 90 / 180 π;
DD = .01 (*m*);
L = 1 (*m*);
G = 250 (*kg/m^2.s*);
W = G π (DD^2/4);
A = π DD^2/4 (*m^2*);
peri = π DD (*m*); DF = 4 A/peri (*m*);
vfg = vg - vf;
ReyNum = G DF / μf;
```

```
x[z_] := Piecewise[{{0, z < zxe0}, {xe[z], z > zxe0 && z < zxe1}, {Min[xe[L], 1], z > zxe1}}]
xp[z_] := x'[z]
Plot[x[z], {z, 0, L}, Frame → True, FrameLabel → {"z [m]", "Quality x"}, GridLines → Automatic, LabelStyle → (FontSize → 16)]
```




$$x = \begin{cases} 0 & x_e < 0 \\ x_e & 0 \le x_e \le 1 \\ 1 & x_e > 1 \end{cases}$$

# Pressure Drop in Two Phase Homogeneous Equilibrium Model

- Solving for $-dP/dz$

$$W\frac{du}{dz} = -A\frac{dp}{dz} - \tau_F P_F - \bar{\rho} g A \sin\theta$$

From Conservation of Momentum

$$-\frac{dp}{dz} = \underbrace{\frac{\tau_F P_F}{A}}_{\substack{\text{Frictional} \\ -\left.\frac{dp}{dz}\right|_F}} + \underbrace{\frac{W}{A}\frac{du}{dz}}_{\substack{\text{Acceleration} \\ -\left.\frac{dp}{dz}\right|_A}} + \boxed{\underbrace{\bar{\rho} g \sin\theta}_{\substack{\text{Gravitational.} \\ -\left.\frac{dp}{dz}\right|_G}}}$$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

- Frictional Pressure Drop
  - Follow along the line of a single fluid
  - $f_{TP}$ stands for the friction coefficient for two phase

$$-\frac{dp}{dz}\bigg|_F = \frac{\tau_f P_F}{A} \qquad \text{Given} \qquad D_H = \frac{4A}{P} \qquad P = \frac{4A}{D_H}$$

$$\Rightarrow -\frac{dp}{dz} = \frac{\tau_f 4\cancel{A}}{D_F \cancel{A}} = \frac{4}{D_F}\left(f_{TF}\frac{1}{2}\bar{\rho}u^2\right)$$

$$\Rightarrow \boxed{-\frac{dp}{dz} = \frac{2}{D_F} f_{TF} G^2 (v_f + x v_{fg})}$$

- Acceleration Pressure Drop

$$-\frac{dp}{dz}\bigg|_A = W\frac{du}{dz} = \frac{W}{A}\frac{d}{dz}\left(\frac{W}{\bar{\rho}A}\right) = \frac{W^2}{A}\frac{d}{dz}\left(\frac{1}{\bar{\rho}}\right)$$

$$= \frac{W^2}{A}\frac{d}{dz}\left(v_f + x v_{fg}\right) = \boxed{\frac{W^2}{A} v_{fg}\frac{dx}{dz}}$$

- Pressure Drop due to Gravity

$$-\frac{dp}{dz}\bigg|_G = \bar{\rho} g \sin\theta = \boxed{\frac{g\sin\theta}{(v_f + x v_{fg})}}$$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## Observations

- Pressure drop increases drastically with vapor formation
- For Adiabatic Flows (meaning no phase transition, $dx/dz=0$ *which implies that* $-dP/dz|_A=0$
- For horizontal flows, gravity induced pressure drop $-dP/dz|_G=0$

Adiabatic horizontal flows are used to determine the frictional gradient from measurement of the total pressure gradient

## + Two-phase Friction Factor

$$-\frac{dp}{dz}\Big|_F = \frac{2}{D_F} f_{TP} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right)$$

$f_{TP}$
*Two phase friction factor*

By Analogy with single $1\phi$ flow

$$f_{TP} = \frac{c}{\left(\frac{G D_F}{\bar{\mu}}\right)^n} = \underbrace{\frac{c}{\left(\frac{G D_F}{\mu_f}\right)^n}}_{f_{fo}} \left(\frac{\bar{\mu}}{\mu_f}\right)^n$$

$f_{fo}$ ← Liquid only

$$f_{TP} = f_{fo} \cdot \left(\frac{\bar{\mu}}{\mu_f}\right)^n$$

→ Mixture viscosity

$$-\frac{dp}{dz}\Big|_F = \underbrace{\frac{2}{D_F} G^2 v_f f_{fo}}_{-\left(\frac{dp}{dz}\right)_{F_{fo}}} \cdot \frac{f_{TP}}{f_{fo}} \left(1 + x\frac{v_{fg}}{v_f}\right)$$

$$-\left(\frac{dp}{dz}\right)_{F_{fo}} \left(\frac{\bar{\mu}}{\mu_f}\right)^n \left(1 + x\frac{v_{fg}}{v_f}\right)$$

$$-\left(\frac{dp}{dz}\right)_F = -\left(\frac{dp}{dz}\right)_{F_{fo}} \phi_{fo}^2$$

← Two phase friction Multiplier

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

- Pressure Drop Calculations/Constant Two-Phase Friction Factor
  - Use constant $f_{TP}$

$$-\frac{dp}{dz} = \frac{2}{D_F} f_{TP} G^2 v_f \left(1 + x\, v_g/v_f\right)$$

$$\boxed{.0029 < f_{TP} < .005}$$

- Pressure Drop Calculations/Using Two-Phase Viscosity Models
  - Using Viscosity Models

$$-\left(\frac{dp}{dz}\right)_F = \left\{\frac{2}{D_F} f_{fo} G^2 v_f\right\} \phi_{fo}^2$$

$$f_{fo} = \frac{c}{\left[\frac{G D_F}{\mu_f}\right]^n} \; ; \quad \phi_{fo}^2 = \left(\frac{\bar{\mu}}{\mu_f}\right)^n \left(1 + x\frac{v_{fg}}{v_f}\right)$$

McAdams (1942) $\quad \frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f}$

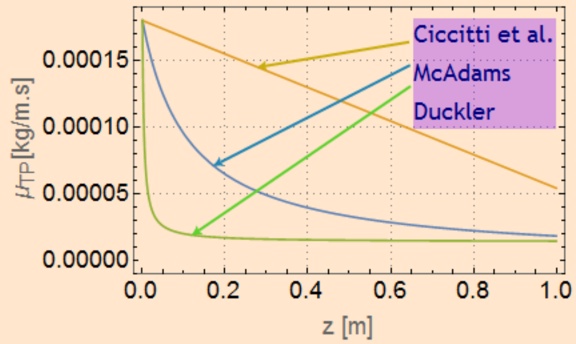
Cicchitti et al (1960) $\quad \bar{\mu} = x\mu_g + (1-x)\mu_f$

Duckler (1964) $\quad \bar{\mu} = \frac{x v_g \mu_g + (1-x) v_f \mu_f}{x v_g + (1-x) v_f}$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## Two-Phase Viscosity Models

```
μMA[z_] := μg μf / (x[z] μf + (1 - x[z]) μg) (*kg/m.s*); "McAdams";
μC[z_] := x[z] μg + (1 - x[z]) μf (*kg/m.s*); "Ciccitti et al.";
μD[z_] := (x[z] vg μg + (1 - x[z]) vf μf) / (x[z] vg + (1 - x[z]) vf) (*kg/m.s*); "Duckler";
Plot[{ μMA[z], μC[z], μD[z]}, {z, 0, 1}, Frame → True, FrameLabel → {"z  [m]", "μTP[kg/m.s]", "", ""}, LabelStyle → (FontSize → 18),
 FrameTicks → Automatic, FrameTicksStyle → Black, GridLines → Automatic, GridLinesStyle → Directive[Dotted, Gray]]
```




```
coef = If[ReyNum < 2300, {c = 16, n = 1}, If[4 × 10^3 < ReyNum < 2 × 10^4, {c = .079, n = .25}],
  If[ReyNum > 2 × 10^4, {c = .046, n = .2}]]
```

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

- Total Two Phase Pressure Drop

$$-\left(\frac{dp}{dz}\right)_{Total} = -\left(\frac{dp}{dz}\right)_{F} + -\left(\frac{dp}{dz}\right)_{A} + -\left(\frac{dp}{dz}\right)_{G}$$

$$= \frac{2}{D_F} f_{TP} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right) + G^2 v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{v_f\left(1 + x\frac{v_{fg}}{v_f}\right)}$$

- Total Pressure Drop

$$\Delta p(z) = \Delta p_{liquid\ phase} + \left[ \int_{z|x_e=0}^{z} \left\{ \frac{2}{D_F} c\left(\frac{G D_F}{\mu_f}\right)^{-n} \left(\frac{\overline{\mu}(\xi)}{\mu_f}\right)^{n} G^2 v_f\left(1 + x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right) + G^2 v_{fg}\left(\frac{dx(\xi)}{dz}\right) + \frac{g\sin(\theta)}{v_f\left(1 + x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right)} \right\} d\xi \right] + \Delta p_{Vapor\ phase}$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

# Example Problem

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*Numerical Example 1: Determination of Pressure Drop using HEM with Constant Two-Phase Friction Factor for Heated Vertical Upflow with Saturated Inlet*

**Saturated water ($x_e = 0$) at mass velocity $G = 250$ kg/m².s and inlet pressure of $p_i = 5$ bar enters a vertical circular tube of diameter $D = 1$ cm and length $L = 100$ cm, where it is subjected to a constant heat flux $q'' = 10^6$ W/m². Neglecting any kinetic or potential energy effects and assuming constant thermophysical properties, use the Homogeneous Equilibrium Model (HEM) with a constant two-phase friction factor $f_{TP} = 0.003$ to determine the following:**

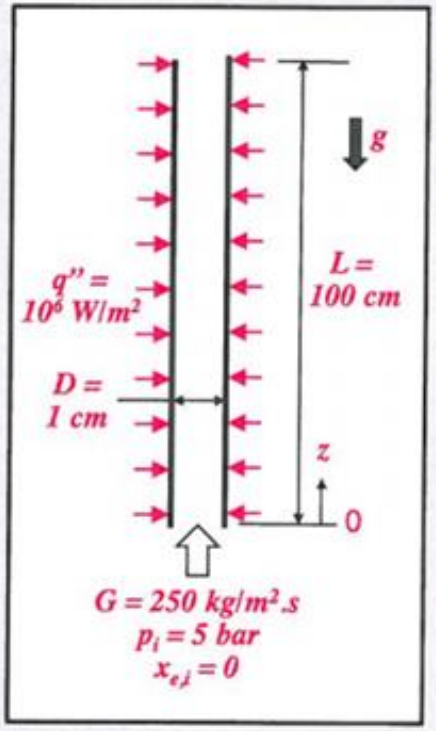
(a) $x_e(z)$, $x_{e,L}$

(b) $\Delta p_F$

(c) $\Delta p_A$

(d) $\Delta p_G$

(e) $\Delta p$

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HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## Forming the Pressure Gradients Integrands

$$\text{intMA}[z\_] := \frac{2}{\text{DF}}\, c \left(\frac{\text{G DF}}{\mu f}\right)^{-n} \left(\frac{\mu\text{MA}[z]}{\mu f}\right)^{n} G^2\, \text{vf} \left(1 + \frac{x[z]\,\text{vfg}}{\text{vf}}\right) + G^2\, \text{vfg}\, \text{xp}[z] + \frac{g\, \text{Sin}[\theta]}{\text{vf}\,(1 + x[z]\, \text{vfg}/\text{vf})};$$

$$\text{intC}[z\_] := \frac{2}{\text{DF}}\, c \left(\frac{\text{G DF}}{\mu f}\right)^{-n} \left(\frac{\mu\text{C}[z]}{\mu f}\right)^{n} G^2\, \text{vf} \left(1 + \frac{x[z]\,\text{vfg}}{\text{vf}}\right) + G^2\, \text{vfg}\, \text{xp}[z] + \frac{g\, \text{Sin}[\theta]}{\text{vf}\,(1 + x[z]\, \text{vfg}/\text{vf})}$$

$$\text{intD}[z\_] := \frac{2}{\text{DF}}\, c \left(\frac{\text{G DF}}{\mu f}\right)^{-n} \left(\frac{\mu\text{D}[z]}{\mu f}\right)^{n} G^2\, \text{vf} \left(1 + \frac{x[z]\,\text{vfg}}{\text{vf}}\right) + G^2\, \text{vfg}\, \text{xp}[z] + \frac{g\, \text{Sin}[\theta]}{\text{vf}\,(1 + x[z]\, \text{vfg}/\text{vf})}$$

## Numerical Integration

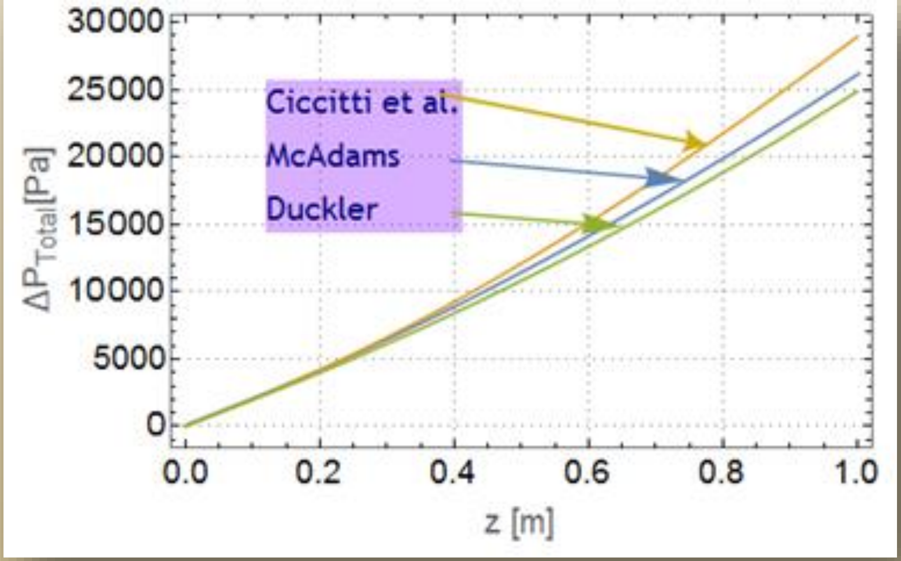
```
ΔPMA[zz_] := NIntegrate[intMA[z], {z, zxe0 + .00001, zz}];
ΔPC[zz_] := NIntegrate[intC[z], {z, zxe0 + .00001, zz}];
ΔPD[zz_] := NIntegrate[intD[z], {z, zxe0 + .00001, zz}];
```

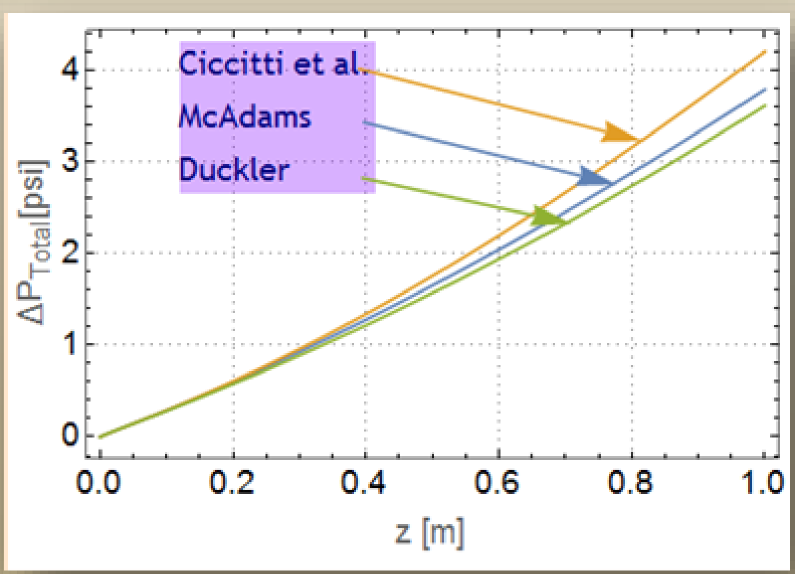
# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

## Plotting ΔP as a Function of z

```
pPa = Plot[{ ΔPMA[z], ΔPC[z], ΔPD[z]}, {z, zxe0, zxe1}, Frame → True, FrameLabel → {"z [m]", "ΔPTotal [Pa]", "", ""}, LabelStyle → (FontSize → 18),
  FrameTicks → Automatic, FrameTicksStyle → Black, GridLines → Automatic, GridLinesStyle → Directive[Dotted, Gray]]

ppsi = Plot[{ ΔPMA[z] / (1.013 × 10^5) 14.7, ΔPC[z] 1 / (1.013 × 10^5) × 14.7, ΔPD[z] / (1.013 × 10^5) 14.7}, {z, zxe0, zxe1}, Frame → True,
  FrameLabel → {"z [m]", "ΔPTotal [psi]", "", ""}, LabelStyle → (FontSize → 18), FrameTicks → Automatic, FrameTicksStyle → Black, GridLines → Automatic,
  GridLinesStyle → Directive[Dotted, Gray]]
```



# Cases Using the Homogenous Equilibrium Model

$$\Delta p(z) = \Delta p_{liquid\ phase} + \left[ \int_{z|x_e=0}^{z} \left\{ \frac{2}{D_F} c\left(\frac{G D_F}{\mu_f}\right)^{-n} \left(\frac{\overline{\mu}(\xi)}{\mu_f}\right)^{n} G^2 v_f\left(1 + x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right) + G^2 v_{fg}\left(\frac{dx(\xi)}{dz}\right) + \frac{g \sin(\theta)}{v_f\left(1+x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right)} \right\} d\xi \right] + \Delta p_{Vapor\ phase}$$

```
p = 5(*bar*);
Cpf = 4312 (*J/kg.K*);
hfg = 2.108 × 10^6 (*J/kg*);
vf = .0011 (*m^3/kg*);
vg = .3748 (*m^3/kg*);
μf = 180.1 × 10^-6 (*kg/m.s*);
μg = 14.06 × 10^-6 (*kg/m.s*);
q = 1.0 × 10^6 (*W/m^2*);
ΔTsub = 30 (*°C*);
g = 9.8 (*m.s^-2*);
θ = 90 / 180 π;
DD = .01 (*m*);
L = 1 (*m*);
G = 250 (*kg/m^2.s*);
W = G π (DD^2/4);
A = π DD^2/4
(*m^2*);
```

```
peri = π DD (*m*); DF = 4 A/peri (*m*);
vfg = vg - vf; ReyNum = G DF / μf;
ReyNumg = G DF / μg;
```

Finding $x_e[z]$ and the z location where the thermodynamic quality $x_e=0$ and $x_e=1$

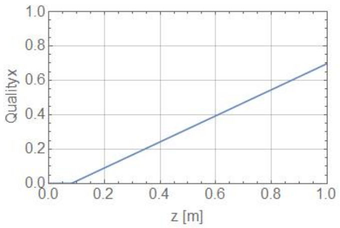
```
xe[z_] := - (Cpf ΔTsub)/hfg + (π DD q)/(W hfg) z
zxe0 = (W Cpf ΔTsub)/(π DD q); L1ph = zxe0;
```

```
xe[L]
```

```
0.697647
```

```
zxe1 = (hfg W)/(π DD q) + (Cpf ΔTsub W)/(π DD q); L2ph = zxe1;
If[L2ph < L, intL = L2ph, intL = L];
```

```
x[z_] :=
  Piecewise[{{0, z < zxe0}, {xe[z], z > zxe0 && z < zxe1},
    {Min[xe[L], 1], z > zxe1}}]
x[z_] :=
  Piecewise[{{0, z < L1ph}, {xe[z], z > L1ph && z < intL},
    {Min[xe[L], 1], z > intL}}]
xp[z_] := x'[z]
Plot[x[z], {z, 0, L + .5 L}, Frame → True,
 FrameLabel → {"z [m]", "Quality x"}, GridLines → Automatic,
 LabelStyle → (FontSize → 16), PlotRange → {{0, L}, {0, 1}}]
```



# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

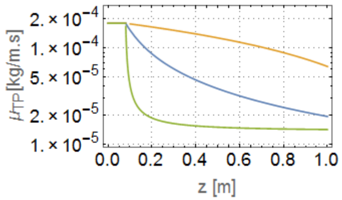
$$\mu MA[z\_] := \frac{\mu g\ \mu f}{x[z]\ \mu f + (1 - x[z])\ \mu g}\ \text{(*kg/m.s*)}; \text{"McAdams"};$$

$$\mu C[z\_] := x[z]\ \mu g + (1 - x[z])\ \mu f\ \text{(*kg/m.s*)};$$

"Ciccitti et al.";

$$\mu D[z\_] := \frac{x[z]\ vg\ \mu g + (1 - x[z])\ vf\ \mu f}{x[z]\ vg + (1 - x[z])\ vf}\ \text{(*kg/m.s*)}; \text{"Duckler"};$$

```
LogPlot[{μMA[z], μC[z], μD[z]}, {z, 0, 1}, Frame → True,
 FrameLabel → {"z [m]", "μTP[kg/m.s]", "", ""},
 LabelStyle → (FontSize → 18), FrameTicks → Automatic,
 FrameTicksStyle → Black, GridLines → Automatic,
 GridLinesStyle → Directive[Dotted, Gray]]
```




```
If[ReyNum < 2300, {c = 16, n = 1}]
If[2.3 × 10^3 < ReyNum < 2 × 10^4, {c = .079, n = .25}]
If[ReyNum > 2 × 10^4, {c = .046, n = .2}]
{0.079, 0.25}

If[ReyNumg < 2300, {c = 16, n = 1}]
If[4 × 10^3 < ReyNumg < 2 × 10^4, {c = .079, n = .25}]
If[ReyNumg > 2 × 10^4, {c = .046, n = .2}]
{0.046, 0.2}
```

$$\Delta p(z) = \Delta p_{liquid\ phase} + \left[\int_{z|x_e=0}^{z}\left\{\frac{2}{D_F}\, c\left(\frac{G D_F}{\mu_f}\right)^{-n}\left(\frac{\overline{\mu}(\xi)}{\mu_f}\right)^{n} G^2 v_f\left(1 + x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right) + G^2 v_{fg}\left(\frac{dx(\xi)}{dz}\right) + \frac{g \sin(\theta)}{v_f\left(1 + x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right)}\right\} d\,\xi\right] + \Delta p_{Vapor\ phase}$$

$$intMA[z\_] := \frac{2}{DF}\, c\left(\frac{G\,DF}{\mu f}\right)^{-n}\left(\frac{\mu MA[z]}{\mu f}\right)^{n} G^2\, vf\left(1 + \frac{x[z]\ vfg}{vf}\right) + G^2\, vfg\, xp[z] + \frac{g\,Sin[\theta]}{vf\ (1 + x[z]\ vfg / vf)};$$

$$intC[z\_] := \frac{2}{DF}\, c\left(\frac{G\,DF}{\mu f}\right)^{-n}\left(\frac{\mu C[z]}{\mu f}\right)^{n} G^2\, vf\left(1 + \frac{x[z]\ vfg}{vf}\right) + G^2\, vfg\, xp[z] + \frac{g\,Sin[\theta]}{vf\ (1 + x[z]\ vfg / vf)}$$

$$intD[z\_] := \frac{2}{DF}\, c\left(\frac{G\,DF}{\mu f}\right)^{-n}\left(\frac{\mu D[z]}{\mu f}\right)^{n} G^2\, vf\left(1 + \frac{x[z]\ vfg}{vf}\right) + G^2\, vfg\, xp[z] + \frac{g\,Sin[\theta]}{vf\ (1 + x[z]\ vfg / vf)}$$

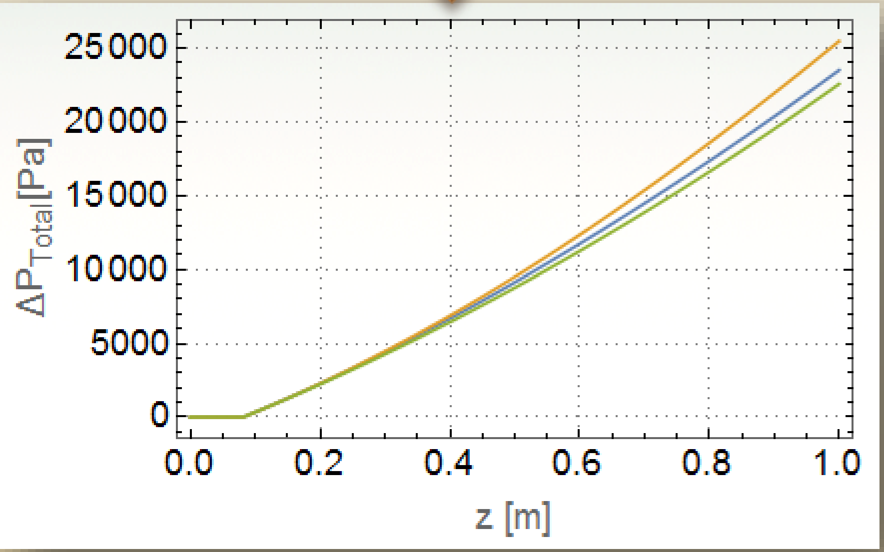
$$\Delta PMA[zz\_] := NIntegrate[intMA[z], \{z, zxe0 + .00001, zz\}] + \frac{2\ c\ ReyNum^{-n}\ vf\ G^2\ L1ph}{DF} + \frac{2\ c\ ReyNumg^{-n}\ vf\ G^2\ (L - intL)}{DF};$$

$$\Delta PC[zz\_] := NIntegrate[intC[z], \{z, zxe0 + .00001, zz\}] + \frac{2\ c\ ReyNum^{-n}\ vf\ G^2\ L1ph}{DF} + \frac{2\ c\ ReyNumg^{-n}\ vf\ G^2\ (L - intL)}{DF};$$

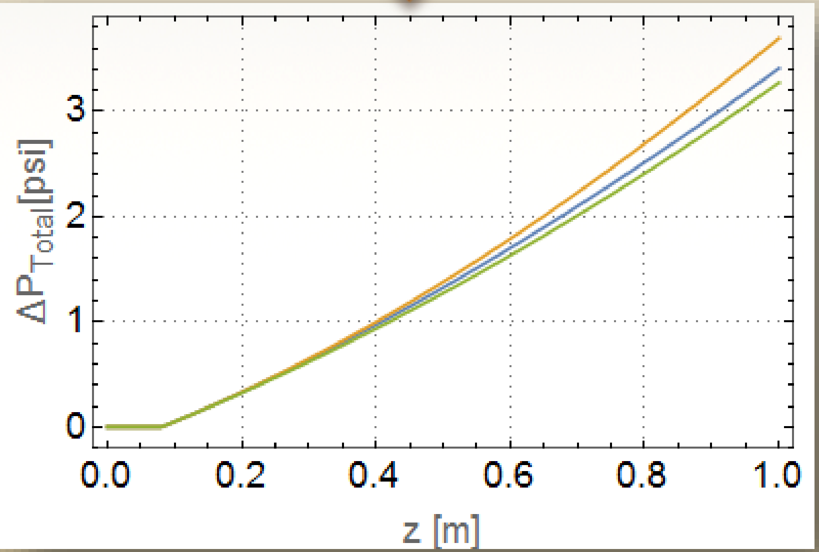
$$\Delta PD[zz\_] := NIntegrate[intD[z], \{z, zxe0 + .00001, zz\}] + \frac{2\ c\ ReyNum^{-n}\ vf\ G^2\ L1ph}{DF} + \frac{2\ c\ ReyNumg^{-n}\ vf\ G^2\ (L - intL)}{DF};$$

# HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

```
Plot[{ΔPMA[z], ΔPC[z], ΔPD[z]}, {z, 0, L}, Frame → True,
 FrameLabel → {"z  [m]", "ΔPTotal [Pa]", "", ""},
 LabelStyle → (FontSize → 18), FrameTicks → Automatic,
 FrameTicksStyle → Black, GridLines → Automatic,
 GridLinesStyle → Directive[Dotted, Gray], PlotRange → All]
```




```
Plot[{ΔPMA[z] / (1.013 × 10^5) 14.7, ΔPC[z] 1 / (1.013 × 10^5) × 14.7,
  ΔPD[z] / (1.013 × 10^5) 14.7}, {z, 0, L}, Frame → True,
 FrameLabel → {"z  [m]", "ΔPTotal [psi]", "", ""},
 LabelStyle → (FontSize → 18), FrameTicks → Automatic,
 FrameTicksStyle → Black, GridLines → Automatic,
 GridLinesStyle → Directive[Dotted, Gray], PlotRange → All]
```

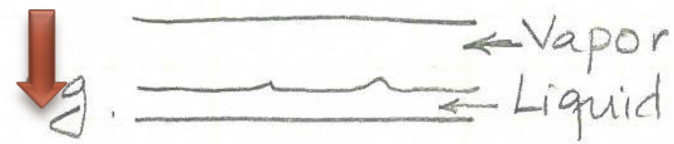


# Pressure Drop in Separated Flows

# Slip Flow Model

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Features
  - Allows for differences in phase velocities
  - Intended for annular and stratified flows.
  - Separate Analyses of individual phases
- Assumptions
  - Different but Uniform phase velocities




$$u_g \neq u_f \qquad S \neq 1$$

  - Uniform pressure over entire flow area

$$p_g = p_f = p$$

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

## Basic Relations

$$u_g = \frac{Q_g}{A_g} \times \frac{\rho_g}{\rho_g} = \frac{W_g}{\rho_g A_g} \qquad \text{but } A_g/A = \alpha \Rightarrow A_g = \alpha A$$

$$\frac{W_g}{W} = x \Rightarrow W_g = x W$$

$$\Rightarrow u_g = \frac{W_g}{\rho_g \alpha A} = \frac{x W}{\rho_g \alpha A} \qquad \frac{W}{A} = G$$

$$u_g = \frac{x G}{\alpha \rho_g}$$

$$u_f = \frac{(1-x) G}{(1-\alpha)\rho_f} \quad \text{from} \quad u_f = \frac{Q_f}{A_f}$$

$$\bar{\rho} = \alpha \rho_g + (1-\alpha)\rho_f \qquad \text{However } S \neq 1$$

In the Homogenous equilibrium model, we derived

$$\frac{1}{\bar{\rho}} = \bar{v} = x v_g + (1-x) v_f \leftarrow \text{This was based on } S = 1$$

$$\Rightarrow \frac{1}{\bar{\rho}} \neq \bar{v} \text{ because } S \neq 1$$

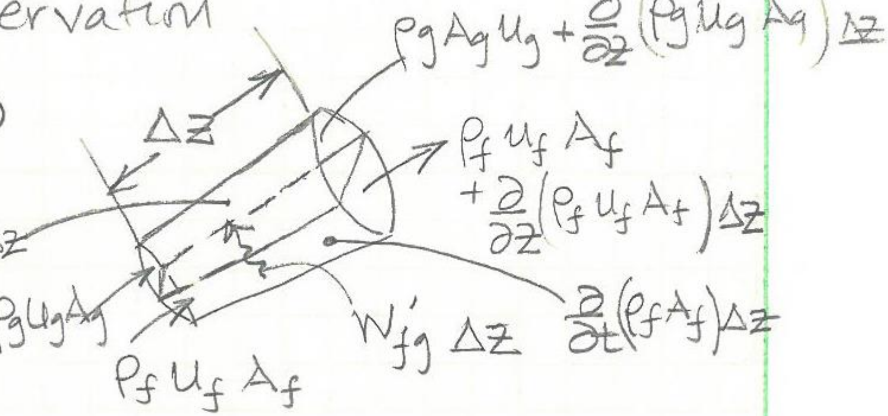
Void fraction becomes an unknown in this formulation

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

## Conservation Laws / Mass Conservation

- Transport Theorem

$$\partial/\partial t \int_V \rho \, dv + \int_S \rho \, \vec{u}_r \cdot d\vec{s} = 0$$




- Vapor phase

$$\frac{\partial}{\partial t}(\rho_g A_g) + \frac{\partial}{\partial z}(\rho_g u_g A_g) - W'_{fg} = 0$$

$$\Rightarrow \frac{\partial}{\partial t}(\rho_g \alpha A) + \frac{\partial}{\partial z}(x W) - W'_{fg} = 0$$

- Liquid phase

$$\frac{\partial}{\partial t}(\rho_f A_f) + \frac{\partial}{\partial z}(\rho_f A_f u_f) + W'_{fg} = 0$$

$$\Rightarrow \frac{\partial}{\partial t}\left[\rho_f (1-\alpha) A\right] + \frac{\partial}{\partial z}\left[(1-x) W\right] + W'_{fg} = 0$$

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Combining the two phases

$$\frac{\partial}{\partial t}\Big[\underbrace{\rho_g \alpha A + \rho_f (1-\alpha) A}_{\bar{\rho} A}\Big] + \frac{\partial}{\partial z}(W) = 0$$

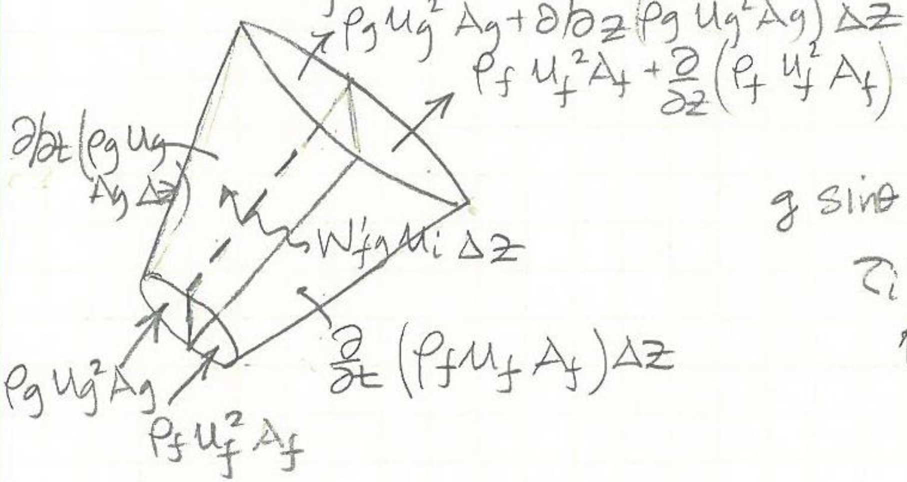
$$\Rightarrow \frac{\partial}{\partial t}\left[\bar{\rho} A\right] + \frac{\partial}{\partial z}(W) = 0$$

$W'_{fg}$ cancelled when combining both phases

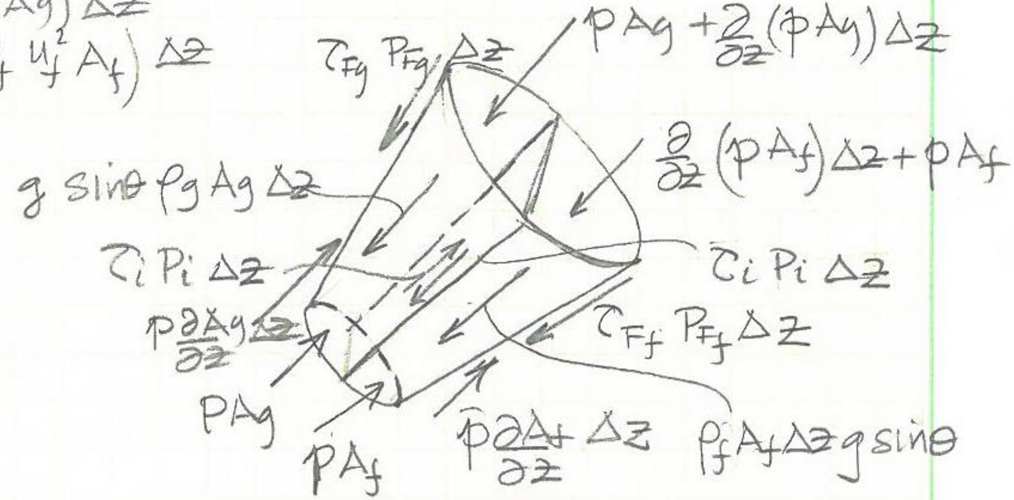
# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

① Momentum Conservation

$$\underbrace{\partial/\partial t \int_V (\rho u)\, dv}_{\text{Rate of Mom. storage}} + \underbrace{\int_S (\rho u)\, \vec{u}_r \cdot d\vec{s}}_{\text{Mom. Out} - \text{Mom. in}} = \underbrace{\sum_i F_z}_{\text{Net force on C.V.}}$$




Conservation of Momentum




Forces

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Vapor phase

$$\partial_t(\rho_g u_g \alpha A) + \partial_z(\rho_g u_g^2 \alpha A) - W'_{fg} u_i = -\alpha A \partial_z p - \tau_{F_g} P_{F_g} - \tau_i P_i - \rho_g \alpha A g \sin\theta$$

- Liquid phase

$$\partial_t(\rho_f u_f (1-\alpha) A) + \partial_z(\rho_f u_f^2 (1-\alpha) A) + W'_{fg} u_i$$
$$= -(1-\alpha) A \partial_z p - \tau_{F_f} P_{F_f} + \tau_i P_i - \rho_f (1-\alpha) A g \sin\theta$$

- Combined

$$\partial_t(W) + \partial_z\left\{\left[\rho_g \alpha u_g^2 + \rho_f (1-\alpha) u_f^2\right] A\right\}$$
$$= -A \partial_z p - \tau_{F_g} P_{F_g} - \tau_{F_f} P_{F_f} - \left[\rho_g \alpha + \rho_f (1-\alpha)\right] A g \sin\theta$$

Interfacial terms cancel out –

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Mass, Energy & Momentum
  - Mass $\partial_t(\bar{\rho}A) + \partial_z(W) = 0$
  - Momentum $\partial_t(W) + \partial_z\left(\left[\rho_g\alpha\left(\frac{xW}{\rho_g\alpha A}\right)^2 + \rho_f(1-\alpha)\left(\frac{(1-x)W}{\rho_f(1-\alpha)A}\right)^2\right]A\right)$

$$= -\partial_z p - \tau_{F_g} P_{F_g} - \tau_{F_f} \cdot P_{F_f} - \bar{\rho} A g \sin\theta$$

- Conservation of Energy.

Internal energy, heat, and work.

$$\partial_t\left[\rho_g h_g^o \alpha A + \rho_f h_f^o (1-\alpha) A\right] + \partial_z\left[x W h_g^o + (1-x) W h_f^o\right] = \left(q_g'' P_{H_g} + q_f'' P_{H_f}\right)$$
$$+ \left[q_g''' \alpha A + q_f'''(1-\alpha) A\right] + \partial_t(pA)$$

Here

$$h_g^o = h_g + u_g^2/2 + g z \sin\theta \qquad h_f^o = h_f + \frac{u_f^2}{2} + g z \sin\theta$$

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Steady State and other simplifications.

Steady state $\partial/\partial t\,(\;) = 0 \Rightarrow$ Continuity yields

$\Rightarrow \partial_z W = 0 \quad \Rightarrow W = \text{const} = GA$ . with $A$ const

$\Rightarrow G = \text{constant}$

Neglecting kinetic and potential energy

$h_k^0 \rightarrow h_k \qquad k = f, g \qquad x = x_e$

Momentum

$$G^2 \frac{d}{dz}\left(\frac{x^2}{\rho_g \alpha} + \frac{(1-x)^2}{\rho_f(1-\alpha)}\right) = -\frac{dp}{dz} - \frac{\tau_{F_g} P_{F_g}}{A} - \frac{\tau_{F_f} P_{F_f}}{A} - \bar{\rho} g \sin\theta$$

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

Energy

$$W \frac{dh}{dz} = q'' P_H \qquad h = h_f + x_e h_{fg}$$

$$q'' P_H = q''_g P_{Hg} + q''_f P_{Hf}$$

$$\Rightarrow \frac{dx_e}{dz} = \frac{q'' P_H}{W h_{fg}}$$

$$\Rightarrow x_e(z) = x_{e,i} + \frac{P_H}{W h_{fg}} \int_0^z q'' d\xi$$

Energy yielded quality as a function of $z$

We know $x[z]$

$\Rightarrow$ In the momentum equation, $\alpha$ and $\tau_F P_F$ are the unknowns

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

As in the previous formulation,

$$-\frac{dp}{dz} = -\frac{dp}{dz}\Big|_F + -\frac{dp}{dz}\Big|_{Acc} + -\frac{dp}{dz}\Big|_G$$

- Frictional

$$-\frac{dp}{dz}\Big|_F = \frac{\tau_F P_F}{A} = \left(\frac{2}{D_F} f_{fo} v_f G^2\right) \phi_{fo}^2$$

Liquid based only.

$$-\frac{dp}{dz}\Big]_G = \bar{\rho} g \sin\theta = \left(\alpha \rho_g + (1-\alpha)\rho_f\right) g \sin\theta$$

$$-\frac{dp}{dz}\Big)_A = G^2 \frac{d}{dz}\left(\frac{x^2 v_g}{\alpha} + \frac{(1-x)\, v_f}{(1-\alpha)}\right)$$

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Gravity

$$-\frac{dp}{dz}\Big|_G = \bar{\rho} g \sin\theta = \left[\alpha \rho_g + (1-\alpha)\rho_f\right] g \sin\theta$$

- Acceleration

$$-\frac{dp}{dz} = G^2 \left[\frac{d}{dz}\left(\frac{x^2 v_g}{\alpha} + \frac{(1-x)^2 v_f}{1-\alpha}\right)\right]$$

When flashing is negligible

$$\Rightarrow \quad x \neq f(p)$$

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

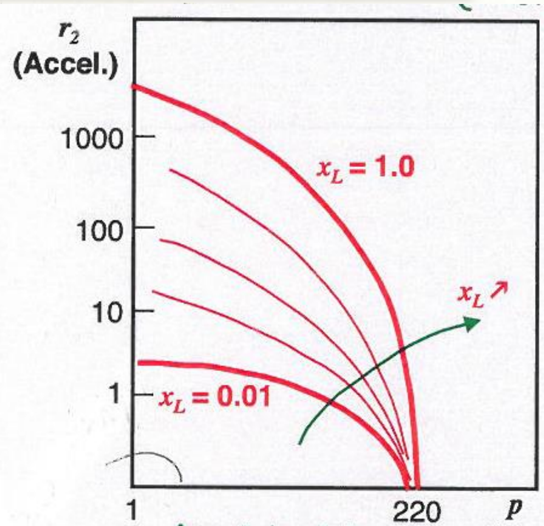
## Martinelli-Nelson Method for Separated Flow Pressure Drop

$$\Delta P = \sum \int \left(-\frac{dp}{dz}\right)_i = \Delta p_F + \Delta p_A + \Delta p_G$$

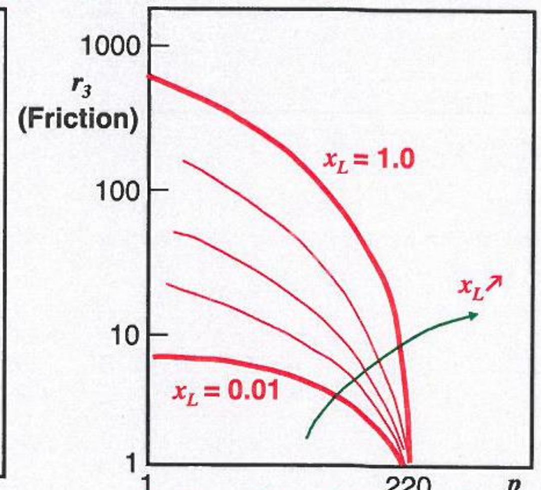
$$\Delta P_A = \left[G^2 v_f\right] r_2$$

$$\Delta P_F = \left[\frac{2L}{D_F} f_{fo} G^2 v_f\right] r_3$$

$$\Delta p_G = \left[\frac{g\,L\,sin\theta}{v_f}\right] r_4$$




- As $x_L$↗, $\Delta p_A$ ↗↗
- As $p$↘, $x_L$ has stronger effect
- $p_{crit} = 220.55$ bar

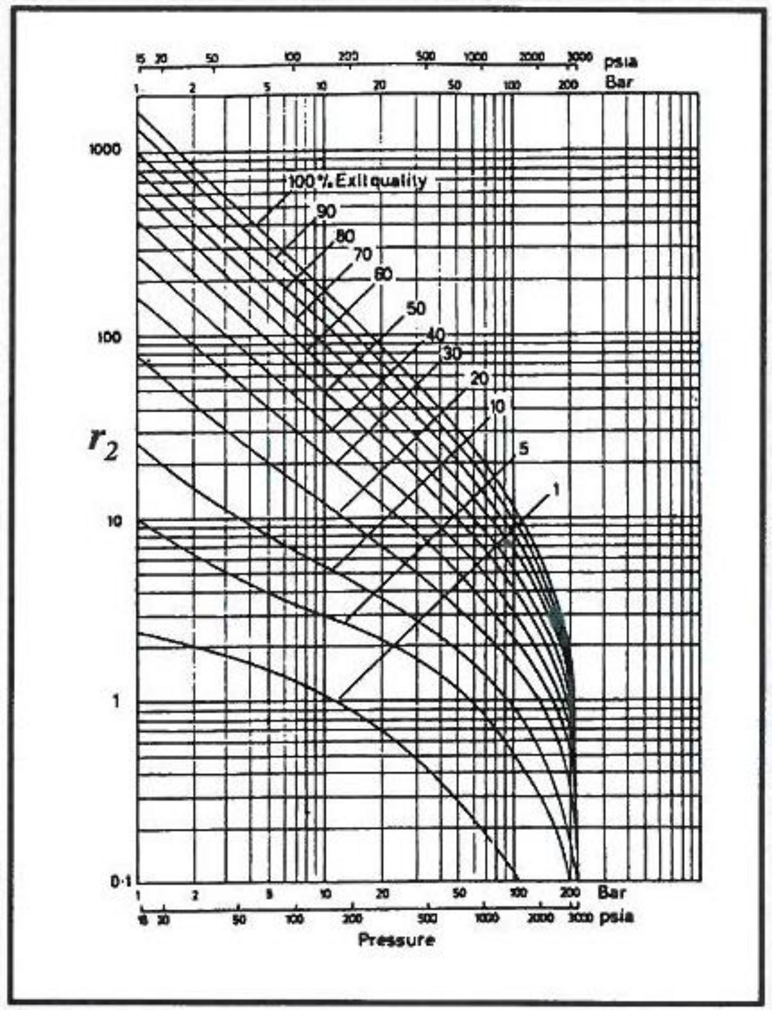


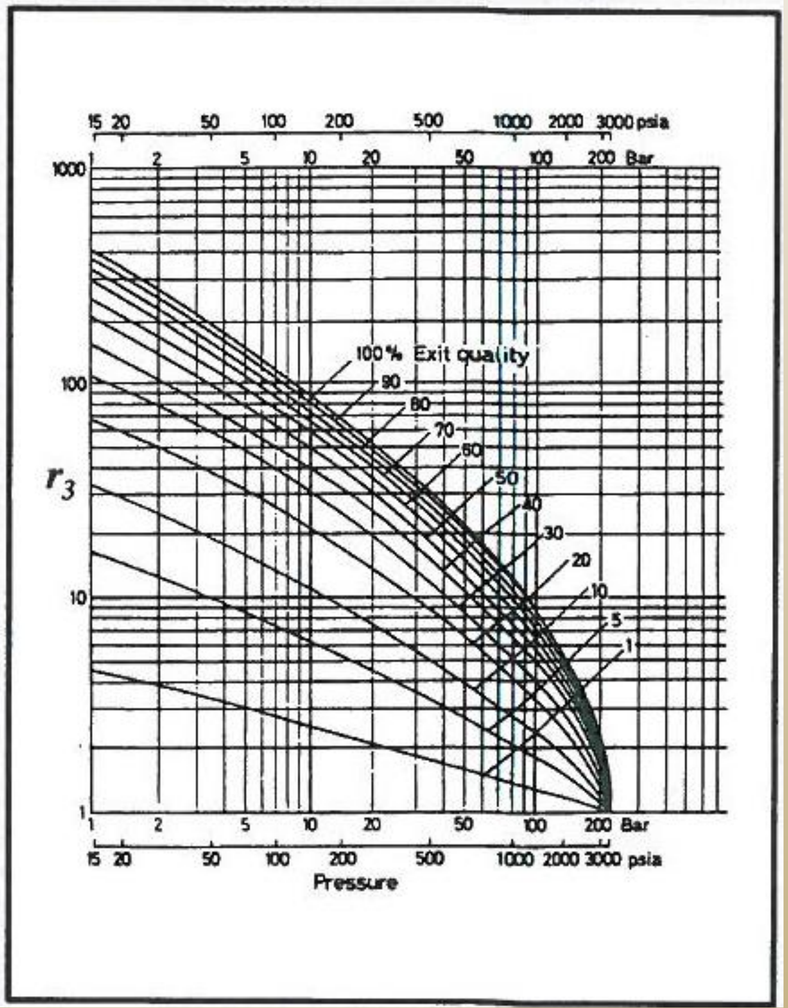

- As $x_L$↗, $\Delta p_F$ ↗↗, especially at low pressures

1. **Martinelli - Nelson (1948)**
   - High $p$
   - Horizontal flow: no $r_4$
   - Steam - water

$$\boxed{r_2, r_3, r_4 = f\left(p_{in}, x_{e,L}\right)}$$

# TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL



# PRESSURE DROP IN SEPARATED FLOWS

Pressure Drop in Separated Flows

Lockhart-Martinelli's Approach for Adiabatic Flows

# PRESSURE DROP IN SEPARATED FLOWS

## Separated Flow Pressure Drop Calculations

Lockhart – Martinelli Method.

- Assumptions
  - Low pressure
  - Horizontal
  - Adiabatic (air-water)
  - $-\left.\frac{dp}{dz}\right]_F$ only

Basis for development of new Correlations by many authors

$$-\left.\frac{dp}{dz}\right]_F = -\left.\frac{dp}{dz}\right|_f \times \phi_f^2 = -\left.\frac{dp}{dz}\right|_g \phi_g^2$$

| | A | n |
|---|---|---|
| Laminar | 16 | 1 |
| Turbulent | .046 | .2 |

$$-\left(\frac{dp}{dz}\right)_f = \frac{2 f_f G^2 (1-x)^2 v_f}{D_F} \; ; \; f_f = \frac{A}{Re_f^n} \; ; \; Re_f = \frac{G(1-x) D_F}{\mu_f}$$

$$-\left(\frac{dp}{dz}\right)_g = \frac{2 f_g G^2 x^2 v_g}{D_F} \; ; \; f_g = \frac{A}{Re_g^n} \; ; \; Re_g = \frac{G x D_F}{\mu_g}$$

# PRESSURE DROP IN SEPARATED FLOWS

○ Lockhart-Martinelli Parameter

$$X^2 = \frac{-\left.\frac{dp}{dz}\right|_f}{-\left.\frac{dp}{dz}\right|_g} \quad : \quad \begin{array}{lll} \text{Laminar} & A = 16 & n = 1 \\ \text{Turbulent} & A = .046 & n = .2 \end{array}$$

○ Sequence of Calculation

– Given $p, x, G, D_F$

- Calculate $-\left.\frac{dp}{dz}\right|_f$ ; $\left.\frac{dp}{dz}\right|_g$
- Calculate $X$
- Determine C from table

| Flow state liquid-gas | C |
|---|---|
| Turbulent – Turbulent | 20 |
| Laminar – Turbulent | 12 |
| Turbulent – Laminar | 10 |
| Laminar – Laminar | 5 |

- $\phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2}$

- $-\left.\frac{dp}{dz}\right|_F = -\left.\frac{dp}{dz}\right|_f \cdot \phi_f^2$
- $\Delta p_F = \int_0^{L_{TP}} \left(-\frac{dp}{dz}\right)_F dz$

# PRESSURE DROP IN SEPARATED FLOWS

## Example Problems

"Fluid is FC-72"

$p = 2$ (*bar*);
$Cpf = 1136$ (*J/kg.K*);
$hfg = 87272$ (*J/kg*);
$vf = .0006515$ (*$m^3/kg$*);
$vg = .0387$ (*$m^3/kg$*);
$\mu f = 349.0 \times 10^{-6}$ (*kg/m.s*);
$\mu g = 12.3 \times 10^{-6}$ (*kg/m.s*);
$\sigma = .0062$ (*N/m*);
$q = 4.0 \times 10^{4}$ (*$W/m^2$*);
$\Delta Tsub = 0$ (*°C*);
$g = 9.8$ (*$m.s^{-2}$*);
$\theta = 0 / 180\ \pi$;
$DD = .005$ (*m*);
$L = .25$ (*m*);
$G = 250$ (*$kg/m^2.s$*);
$W = G\,\pi \left(\frac{DD^2}{4}\right)$;
$A = \frac{\pi\, DD^2}{4}$ (*$m^2$*);
$peri = \pi\, DD$ (*m*);
$DF = \frac{4\,A}{peri}$ (*m*);
$vfg = vg - vf$;
$ReyNum = G\,DF / \mu f$;
$ReyNumg = G\,DF / \mu g$;

## Quality as a Function of z, $x_e(z)$

$xe[z\_] := -\frac{Cpf\ \Delta Tsub}{hfg} + \frac{\pi\ DD\ q}{W\ hfg}\ z$
$xe0 = xe[0]$
$xel = xe[L]$

0.

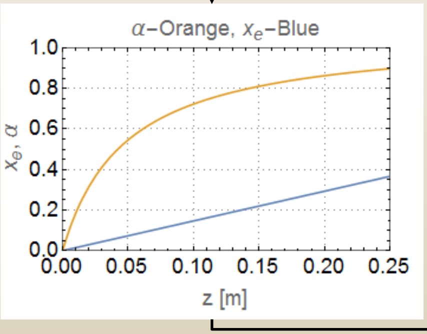
0.36667

$zxe0 = \frac{W\ Cpf\ \Delta Tsub}{\pi\ DD\ q}$; $L1ph = zxe0$;
$zxe1 = \frac{hfg\ W}{\pi\ DD\ q} + \frac{Cpf\ \Delta Tsub\ W}{\pi\ DD\ q}$; $L2ph = zxe1$;
If[L2ph < L, intL = L2ph, intL = L];

## Void Fraction and Quality (Zivi, 1964)

$\alpha[z\_] := \left(1 + \frac{1 - xe[z]}{xe[z]} \left(\frac{vf}{vg}\right)^{2/3}\right)^{-1}$

```
Plot[{xe[z], α[z]}, {z, zxe0 + .00001, intL},
  Frame → True,
  FrameLabel → {"z [m]", "xe, α",
    "α-Orange, xe-Blue", ""},
  LabelStyle → (FontSize → 18),
  FrameTicks → Automatic, FrameTicksStyle → Black,
  GridLines → Automatic,
  GridLinesStyle → Directive[Dotted, Gray],
  PlotRange → {{0, L}, {0, 1}}]
```




## Friction Factors on the liquid and gas sides

$ff[z\_] :=$
$\text{Piecewise}\Big[\Big\{\Big\{16 \left(\frac{G\,(1 - xe[z])\,DD}{\mu f}\right)^{-1}, \left(\frac{G\,(1 - xe[z])\,DD}{\mu f}\right) < 2000\Big\},$
$\Big\{.079 \left(\frac{G\,(1 - xe[z])\,DD}{\mu f}\right)^{-.25}, 2000 < \left(\frac{G\,(1 - xe[z])\,DD}{\mu f}\right) < 20\,000\Big\},$
$\Big\{.046 \left(\frac{G\,(1 - xe[z])\,DD}{\mu f}\right)^{-.2}, \left(\frac{G\,(1 - xe[z])\,DD}{\mu f}\right) > 20\,000\Big\}\Big\}\Big]$

$fg[z\_] :=$
$\text{Piecewise}\Big[\Big\{\Big\{16 \left(\frac{G\,(xe[z])\,DD}{\mu g}\right)^{-1}, \left(\frac{G\,(xe[z])\,DD}{\mu g}\right) < 2000\Big\},$
$\Big\{.079 \left(\frac{G\,(xe[z])\,DD}{\mu g}\right)^{-.25}, 2000 < \left(\frac{G\,(xe[z])\,DD}{\mu g}\right) < 20\,000\Big\},$
$\Big\{.046 \left(\frac{G\,(xe[z])\,DD}{\mu g}\right)^{-.2}, \left(\frac{G\,(xe[z])\,DD}{\mu g}\right) > 20\,000\Big\}\Big\}\Big]$

## Constant $C$

$CC[z\_] :=$
$\text{Piecewise}\Big[\Big\{\Big\{5, \left(\frac{G\,(1 - xe[z])\,DD}{\mu f}\right) < 2000\ \&\&\ \left(\frac{G\,(xe[z])\,DD}{\mu g}\right) < 2000\Big\},$
$\Big\{12, \left(\frac{G\,(1 - xe[z])\,DD}{\mu f}\right) < 2000\ \&\&\ 2000 < \left(\frac{G\,(xe[z])\,DD}{\mu g}\right)\Big\},$
$\Big\{10, 2000 < \left(\frac{G\,(1 - xe[z])\,DD}{\mu f}\right)\ \&\&\ \left(\frac{G\,(xe[z])\,DD}{\mu g}\right) < 2000\Big\},$
$\Big\{20, 2000 < \left(\frac{G\,(1 - xe[z])\,DD}{\mu f}\right)\ \&\&\ 2000 < \left(\frac{G\,(xe[z])\,DD}{\mu g}\right)\Big\}\Big\}\Big]$

# PRESSURE DROP IN SEPARATED FLOWS

## Frictional, Acceleration and Gravitational Pressure Gradients

$$\texttt{dpdzF[z\_]} := \frac{1}{\texttt{DD}}\, 2\, G^2\, \texttt{vf ff[z]}\,(1-\texttt{xe[z]})^2 \left(1+\frac{\texttt{CC[z]}}{\sqrt{\frac{\texttt{vf ff[z]}\,(1-\texttt{xe[z]})^2}{\texttt{vg fg[z] xe[z]}^2}}}+\frac{\texttt{vg fg[z] xe[z]}^2}{\texttt{vf ff[z]}\,(1-\texttt{xe[z]})^2}\right)$$

$$\texttt{dpdzA[z\_]} := G^2\, \texttt{vf}\left(\frac{(\texttt{xe[z]})^2\, \texttt{vg}}{\alpha\texttt{[z] vf}}+\frac{(1-(\texttt{xe[z]}))^2}{(1-\alpha\texttt{[z]})}-1\right)$$

$$\texttt{dpdzG[z\_]} := \left(\frac{\alpha\texttt{[z]}}{\texttt{vg}}+\frac{(1-\alpha\texttt{[z]})}{\texttt{vf}}\right) \texttt{g Sin[}\theta\texttt{]}$$

## Integrated Pressure Drop, $\Delta P = \int_0^L -(dp/dz)\, dz$

```
ΔPF[z_] := NIntegrate[dpdzF[zz], {zz, zxe0, z}]
```

$$\texttt{ΔPA[z\_]} := G^2\, \texttt{vf}\left(\frac{(\texttt{xe[z]})^2\, \texttt{vg}}{\alpha\texttt{[z] vf}}+\frac{(1-(\texttt{xe[z]}))^2}{(1-\alpha\texttt{[z]})}-1\right)$$

```
ΔPG[z_] := NIntegrate[dpdzG[zz], {zz, zxe0, z}]

Plot[{ΔPF[z], ΔPA[z], ΔPG[z], ΔPF[z] + ΔPA[z] + ΔPG[z]},
 {z, zxe0 + .00001, intL}, Frame → True,
 FrameLabel → {"z [m]", "ΔPF, ΔPA, ΔPG, ΔPT [Pa]",
   "ΔPF-Blue, ΔPA-Orange, ΔPG-Green, ΔPT-Red", ""},
 LabelStyle → (FontSize → 18), FrameTicks → Automatic,
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 GridLinesStyle → Directive[Dotted, Gray], PlotRange → All]
```
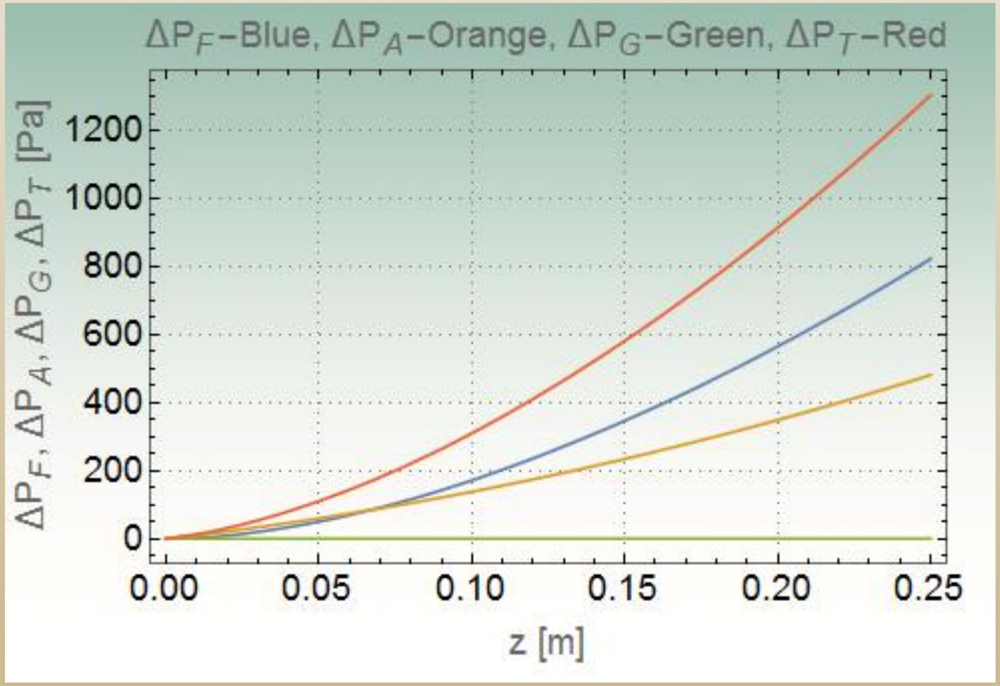
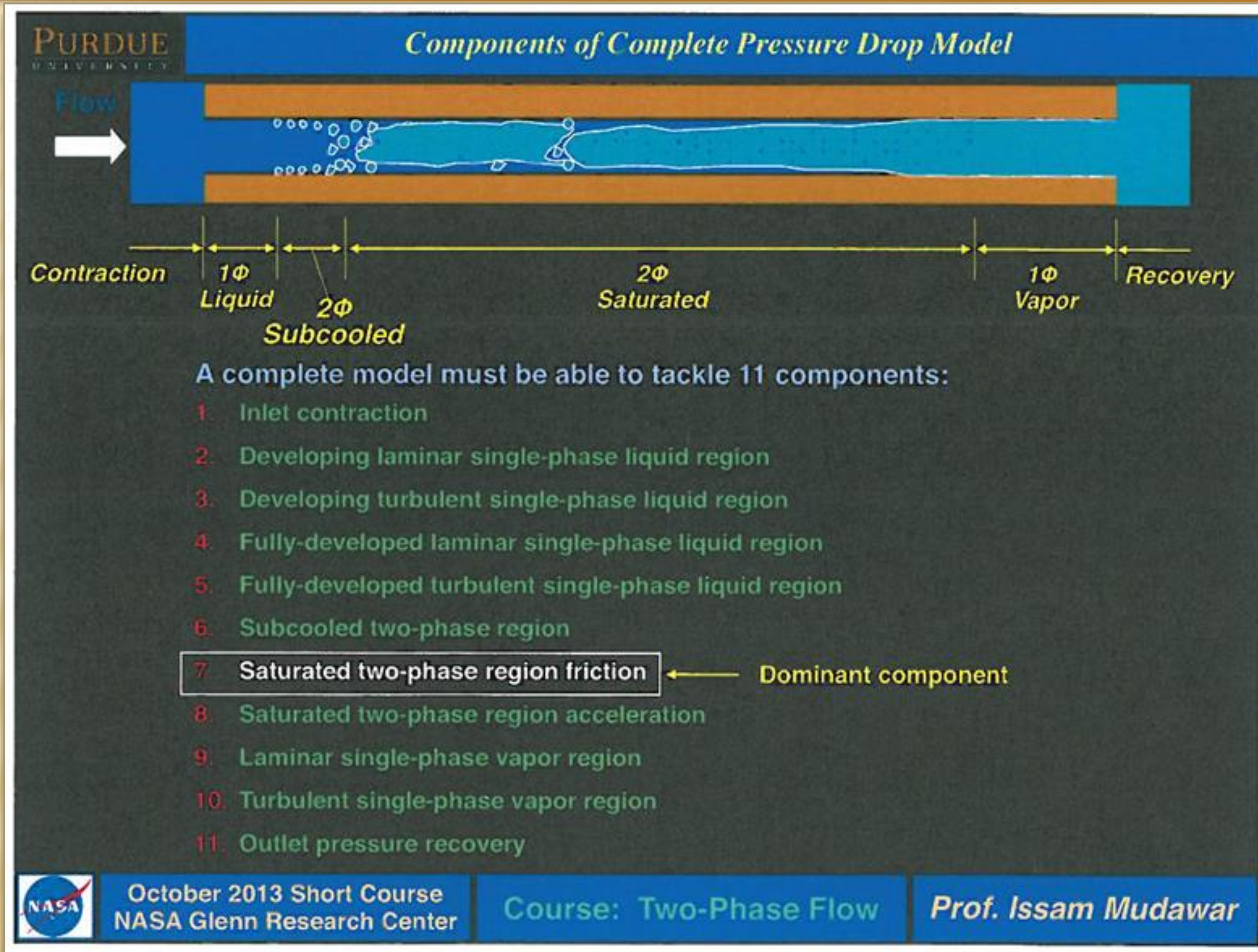



# PRESSURE DROP IN SEPARATED FLOWS

Pressure Drop in Separated Flows

SFM with Mudawar's Universal Evaporating Flows Correlation

# PRESSURE DROP IN SEPARATED FLOWS



# PRESSURE DROP IN SEPARATED FLOWS

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## More Recent Efforts

- Most published correlations for two-phase pressure drop recommended for relatively large tube diameters. Popular and successful correlations include those of Friedel (1979) and Müller-Steinhagen & Heck (1986)
- Small diameters crucial to reducing TCS mass in space systems and ensuring gravity independent evaporation and condensation
- New efforts undertaken at Purdue University Boiling and Two-Phase Flow Lab (PU-BTPFL) to derive universal correlations for small diameters (less than ~ 6 mm) by amassing published data for many fluids and over very broad ranges of operating conditions for:
  - Adiabatic and condensing flows
  - Evaporating flows
- The Purdue correlations are being tested against newly obtained microgravity data

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## Limitations of Two-Phase Flow and Heat Transfer Correlations

- **One-phase: forced convection in a pipe:**

$$Nu = 0.023 Re^{0.8} Pr^{1/3} \qquad \begin{cases} Re > 10{,}000 \\ 0.6 < Pr < 160 \end{cases}$$

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

**Very powerful correlation applicable to many fluids over very broad range of flow conditions**

- **Two-phase: steam-water critical heat flux in a pipe:**

$$\frac{q''_m}{G h_{fg}} = f\left(\frac{\rho_f}{\rho_g}, \frac{G^2 L}{\sigma \rho_f}, \frac{c_{p,f} \Delta T_{sub}}{h_{fg}}, \frac{L}{D}, \frac{G}{\rho_f \sqrt{g D}}, \ldots\right)$$

$$\Pi_1 = f(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6, \ldots)$$

$$\begin{cases} \Pi_{2,min} < \Pi_2 < \Pi_{2,max} \\ \Pi_{3,min} < \Pi_3 < \Pi_{3,max} \\ \Pi_{4,min} < \Pi_4 < \Pi_{4,max} \\ \Pi_{5,min} < \Pi_5 < \Pi_{5,max} \\ \Pi_{6,min} < \Pi_6 < \Pi_{6,max} \end{cases}$$

**Simultaneously satisfying ranges for several parameters greatly limits overall usefulness of correlation … Correlations cannot be extended with confidence to other fluids and/or beyond their validity range!**

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# PRESSURE DROP IN SEPARATED FLOWS

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## Dimensionless Groups Employed by Various Investigators in Prediction of Two-Phase Pressure Gradient

| | | |
|---|---|---|
| Liquid- or gas-only Reynolds number: | $Re_{fo} = \dfrac{G D_h}{\mu_f}$ , $Re_{go} = \dfrac{G D_h}{\mu_g}$ | $\dfrac{\text{Inertia}}{\text{Viscous force}}$ |
| Superficial liquid or gas Reynolds number: | $Re_f = \dfrac{G(1-x) D_h}{\mu_f}$ , $Re_g = \dfrac{G x D_h}{\mu_g}$ | $\dfrac{\text{Inertia}}{\text{Viscous force}}$ |
| Density ratio: | $\dfrac{\rho_f}{\rho_g}$ | $\dfrac{\text{Liquid density}}{\text{Vapor density}}$ |
| Weber Number: | $We = \dfrac{G^2 D_h}{\rho_f \sigma}$ | $\dfrac{\text{Inertia}}{\text{Surface tension force}}$ |
| Capillary number: | $Ca = \dfrac{\mu_f G}{\rho_f \sigma} \left( = \dfrac{We}{Re_{fo}} \right)$ | $\dfrac{\text{Viscous force}}{\text{Surface tension force}}$ |
| Liquid- or gas-only Suratman number: | $Su_{fo} = \dfrac{\rho_f \sigma D_h}{\mu_f^2} \left( = \dfrac{Re_{fo}^2}{We} \right)$ , $Su_{go} = \dfrac{\rho_g \sigma D_h}{\mu_g^2} \left( = \dfrac{Re_{go}^2}{We} \right)$ | - |
| Froude number: | $Fr = \dfrac{G^2}{g D_h \rho_f^2}$ | $\dfrac{\text{Inertia}}{\text{Body force}}$ |
| Bond Number: | $Bd = \dfrac{g(\rho_f - \rho_g) D_h^2}{\sigma}$ | $\dfrac{\text{Bouyancy force}}{\text{Surface tension force}}$ |
| Confinement Number: | $N_{conf} = \sqrt{\dfrac{\sigma}{g(\rho_f - \rho_g) D_h^2}} \left( = \sqrt{\dfrac{1}{Bd}} \right)$ | $\sqrt{\dfrac{\text{Surface tension force}}{\text{Body force}}}$ |
| Galileo Number: | $Ga = \dfrac{\rho_f g(\rho_f - \rho_g) D_h^3}{\mu_f^2}$ | - |

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## Pressure Drop in Saturated Two-Phase Flow Region

**Two-phase pressure drop:** $\Delta p_{tp} = \Delta p_F + \Delta p_G + \Delta p_A$

**Accelerational pressure drop:**

$$-\left(\frac{dp}{dz}\right)_A = G^2 \frac{d}{dz}\left[\frac{v_g x^2}{\alpha} + \frac{v_f (1-x)^2}{(1-\alpha)}\right] \quad \text{where} \quad \alpha = \left[1 + \left(\frac{1-x_e}{x_e}\right)\left(\frac{v_f}{v_g}\right)^{2/3}\right]^{-1}$$

(Zivi, 1964)

$\Delta p_A > 0$ for boiling flows

$\Delta p_A < 0$ for condensing flows

**Gravitational pressure drop:**

$$-\left(\frac{dp}{dz}\right)_G = \left[\alpha \rho_g + (1-\alpha)\rho_f\right] g \sin\phi$$

**Frictional pressure drop:**

Homogeneous Equilibrium Model (HEM)

$$-\left(\frac{dp}{dz}\right)_F = \frac{2 f_{tp} \bar{\rho} \bar{u}^2}{D_h} = \frac{2 f_{tp} v_f G^2}{D_h}\left(1 + x \frac{v_{fg}}{v_f}\right)$$

$f_{tp} = 16 Re_{tp}^{-1}$ for $Re_{tp} < 2{,}000$

$f_{tp} = 0.079 Re_{tp}^{-0.25}$ for $2{,}000 \le Re_{tp} < 20{,}000$

$f_{tp} = 0.046 Re_{tp}^{-0.2}$ for $Re_{tp} \ge 20{,}000$

where $Re_{tp} = \dfrac{G D_h}{\mu_{tp}}$

Separated Flow Model (SFM)

$$\left(\frac{dp}{dz}\right)_F = \left(\frac{dp}{dz}\right)_f \phi_f^2 \quad \text{where} \quad \phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2}, \quad X^2 = \frac{(dp/dz)_f}{(dp/dz)_g}$$

$$-\left(\frac{dp}{dz}\right)_f = \frac{2 f_f v_f G^2 (1-x)^2}{D_h}, \quad -\left(\frac{dp}{dz}\right)_g = \frac{2 f_g v_g G^2 x^2}{D_h}$$

$f_k = 16 Re_k^{-1}$ for $Re_k < 2{,}000$

$f_k = 0.079 Re_k^{-0.25}$ for $2{,}000 \le Re_k < 20{,}000$

$f_k = 0.046 Re_k^{-0.2}$ for $Re_k \ge 20{,}000$ where $k = f$ or $g$

**Two-phase pressure drop:**

$$\Delta p_{tp} = \int_0^{L_{tp}} \left[-\left(\frac{dp}{dz}\right)_F - \left(\frac{dp}{dz}\right)_G - \left(\frac{dp}{dz}\right)_A\right] dz$$

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# PRESSURE DROP IN SEPARATED FLOWS

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## *New PU-BTPFL Two-Phase Frictional Pressure Drop Correlation for Evaporating Flow in Small Diameter Tubes*

**Consolidated database: 2378 boiling pressure drop data points from 16 sources**

- Working fluids: R12, R134a, R22, R245fa, R410A, FC-72, ammonia, CO2, water
- Hydraulic diameter: $0.349 < D_h < 5.35$ mm
- Mass velocity: $33 < G < 2738$ kg/m²s
- Liquid-only Reynolds number: $156 < Re_{fo} < 28\,010$
- Superficial liquid Reynolds number: $0 < Re_f < 16\,020$
- Superficial vapor (or gas) Reynolds number: $0 < Re_g < 199\,500$
- Flow quality: $0 < x < 1$
- Reduced pressure: $0.005 < P_R < 0.78$

$$\left(\frac{dp}{dz}\right)_F = \left(\frac{dp}{dz}\right)_f \phi_f^2 \quad \text{where} \quad \phi_f^2 = 1 + \frac{\boxed{C}}{X} + \frac{1}{X^2}, \quad X^2 = \frac{(dp/dz)_f}{(dp/dz)_g}$$

$$-\left(\frac{dp}{dz}\right)_f = \frac{2 f_f v_f G^2 (1-x)^2}{D_h}, \quad -\left(\frac{dp}{dz}\right)_g = \frac{2 f_g v_g G^2 x^2}{D_h}$$

$f_k = 16 Re_k^{-1}$ for $Re_k < 2{,}000$

$f_k = 0.079 Re_k^{-0.25}$ for $2{,}000 \le Re_k < 20{,}000$

$f_k = 0.046 Re_k^{-0.2}$ for $Re_k \ge 20{,}000$ where $k = f$ or $g$

for laminar flow in rectangular channel,

$$f_k Re_k = 24\left(1 - 1.3553\beta + 1.9467\beta^2 - 1.7012\beta^3 + 0.9564\beta^4 - 0.2537\beta^5\right)$$

$$Re_f = \frac{G(1-x)D_h}{\mu_f}, \quad Re_g = \frac{G x D_h}{\mu_g}, \quad Re_{fo} = \frac{G D_h}{\mu_f}, \quad Su_{go} = \frac{\rho_g \sigma D_h}{\mu_g^2}$$

$$C = C_{non-boiling}\left[1 + 530\, We_{fo}^{0.52}\left(Bo\,\frac{P_H}{P_F}\right)^{1.09}\right] \quad \text{for} \quad Re_f < 2000$$

$$C = C_{non-boiling}\left[1 + 60\, We_{fo}^{0.32}\left(Bo\,\frac{P_H}{P_F}\right)^{0.78}\right] \quad \text{for} \quad Re_f \ge 2000$$

where $We_{fo} = \frac{G^2 D_h}{\rho_f \sigma}$, $Bo = \frac{q''_H}{G h_{fg}}$

$q''_H$ effective heat flux averaged over heated perimeter of channel

$P_H$ heated perimeter of channel

$P_F$ wetted perimeter of channel

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# PRESSURE DROP IN SEPARATED FLOWS

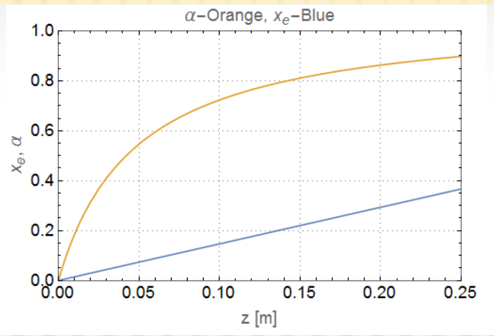
## Example Problems

```
"Fluid is FC-72"

p = 2 (*bar*);
Cpf = 1136 (*J/kg.K*);
hfg = 87272 (*J/kg*);
vf = .0006515 (*m^3/kg*);
vg = .0387 (*m^3/kg*);
μf = 349.0 × 10^-6 (*kg/m.s*);
μg = 12.3 × 10^-6 (*kg/m.s*);
σ = .0062 (*N/m*);
q = 4.0 × 10^4 (*W/m^2*);
ΔTsub = 0 (*°C*);
g = 9.8 (*m.s^-2*);
θ = 0 / 180 π;
DD = .005 (*m*);
L = .25 (*m*);
G = 250 (*kg/m^2.s*);
W = G π (DD^2/4);

A = π DD^2/4 (*m^2*); peri = π DD (*m*);
DF = 4 A/peri (*m*);
vfg = vg - vf;
ReyNum = G DF / μf;
ReyNumg = G DF / μg;
BoNum = q/(G hfg)
WeNumf0 = G^2 DF vf/σ

xe[z_] := -Cpf ΔTsub/hfg + π DD q/(W hfg) z
xe0 = xe[0];
xel = xe[L];

zxe0 = W Cpf ΔTsub/(π DD q); L1ph = zxe0;
zxe1 = hfg W/(π DD q) + Cpf ΔTsub W/(π DD q); L2ph = zxe1;
If[L2ph < L, intL = L2ph, intL = L];
```

## Void Fraction and Quality (Zivi, 1964)

```
α[z_] := (1 + (1 - xe[z])/xe[z] (vf/vg)^(2/3))^-1
PF[z_] := π DD (1 - α[z]); PF[z_] := π DD;
PH = π DD
0.015708
```

```
Plot[{xe[z], α[z]}, {z, zxe0 + .00001, intL}, Frame → True,
  FrameLabel → {"z [m]", "xe, α", "α-Orange, xe-Blue", ""},
  LabelStyle → (FontSize → 18), FrameTicks → Automatic,
  FrameTicksStyle → Black, GridLines → Automatic,
  GridLinesStyle → Directive[Dotted, Gray],
  PlotRange → {{0, L}, {0, 1}}]
```




## Friction Factors on the Liquid and Gas Sides

```
ff[z_] :=
 Piecewise[{{16 (G (1 - xe[z]) DD/μf)^-1, (G (1 - xe[z]) DD/μf) < 2000},
   {.079 (G (1 - xe[z]) DD/μf)^-.25, 2000 < (G (1 - xe[z]) DD/μf) < 20000},
   {.046 (G (1 - xe[z]) DD/μf)^-.2, (G (1 - xe[z]) DD/μf) > 20000}}]
fg[z_] :=
 Piecewise[{{16 (G (xe[z]) DD/μg)^-1, (G (xe[z]) DD/μg) < 2000},
   {.079 (G (xe[z]) DD/μg)^-.25, 2000 < (G (xe[z]) DD/μg) < 20000},
   {.046 (G (xe[z]) DD/μg)^-.2, (G (xe[z]) DD/μg) > 20000}}]
```

## Constant $C_{\text{Non-Boiling}}$

```
CC[z_] :=
 Piecewise[{{5, (G (1 - xe[z]) DD/μf) < 2000 && (G (xe[z]) DD/μg) < 2000},
   {12, (G (1 - xe[z]) DD/μf) < 2000 && 2000 < (G (xe[z]) DD/μg)},
   {10, 2000 < (G (1 - xe[z]) DD/μf) && (G (xe[z]) DD/μg) < 2000},
   {20, 2000 < (G (1 - xe[z]) DD/μf) && 2000 < (G (xe[z]) DD/μg)}}]
```

## Constant $C_{\text{Boiling}}$

```
CCM[z_] :=
 Piecewise[
  {{CC[z] (1 + 530 WeNumf0^.52 (BoNum PH / PF[z])^1.09),
    (G (1 - xe[z]) DD/μf) < 2000},
   {CC[z] (1 + 60 WeNumf0^.32 (BoNum PH / PF[z])^.78),
    (G (1 - xe[z]) DD/μf) > 2000}}]
```

## Frictional, Acceleration and Gravitational Pressure Gradients

```
dpdzF[z_] :=
 1/DD 2 G^2 vf ff[z] (1 - xe[z])^2
  (1 + CCM[z]/Sqrt[(vf ff[z] (1-xe[z])^2)/(vg fg[z] xe[z]^2)] + (vg fg[z] xe[z]^2)/(vf ff[z] (1 - xe[z])^2))

dpdzA[z_] := G^2 vf ((xe[z])^2 vg/(α[z] vf) + (1 - (xe[z]))^2/(1 - α[z]) - 1)

dpdzG[z_] := (α[z]/vg + (1 - α[z])/vf) g Sin[θ]
```

## Integrated Pressure Drop, $\Delta P = \int_0^L -(dp/dz)\,dz$

```
ΔPF[z_] := NIntegrate[dpdzF[zz], {zz, xe0, z}]
ΔPA[z_] := G^2 vf ((xe[z])^2 vg/(α[z] vf) + (1 - (xe[z]))^2/(1 - α[z]) - 1)
ΔPG[z_] := NIntegrate[dpdzG[zz], {zz, zxe0, z}]
```

# PRESSURE DROP IN SEPARATED FLOWS

```
Plot[{ΔPF[z], ΔPA[z], ΔPG[z], ΔPF[z] + ΔPA[z] + ΔPG[z]},
 {z, zxe0 + .00001, intL}, Frame → True,
 FrameLabel → {"z [m]", "ΔP_F, ΔP_A, ΔP_G, ΔP_T [Pa]",
   "ΔP_F-Blue, ΔP_A-Orange, ΔP_G-Green, ΔP_T-Red", ""},
 LabelStyle → (FontSize → 18), FrameTicks → Automatic,
 FrameTicksStyle → Black, GridLines → Automatic,
 GridLinesStyle → Directive[Dotted, Gray], PlotRange → All]
```